Lekcija VI
Elektromagnetni talasi

UVOD U MATEMATIČKI FORMALIZAM

Uvod: 2 Polja i 3 Operacije

Skalar: veličina koja se može prikazatia pozitivnim ili negativnim brojem (masa, temperatura, energija, pritisak)

$$x$$

Vektor: veličina koja osim brojne vrednosti ima i pravac u prostoru (Sila, brzina, ubrzanje)

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

Suma: dodavanje komponenti

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + \left(A_y + B_y\right)\hat{j} + (A_z + B_z)\hat{k}$$

Proizvod: vektor puta skalar

$$c\vec{A} = cA_x \hat{i} + cA_y \hat{j} + cA_z \hat{k}$$

Skalarni proizvod: pomnožiti komponente i sabrati

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$$

Vektorski proizvod: determinanta sa jediničnim vektorima

$$\vec{A}\times\vec{B}=\det\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{bmatrix}$$

$$=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$

Skalarno polje: skalarna veličina definisana u svakoj tački 2D ili 3D prostora

Analitički

$$S(x, y) = f(x, y)$$

npr:

$$S(x, y) = x\sin(xy)$$

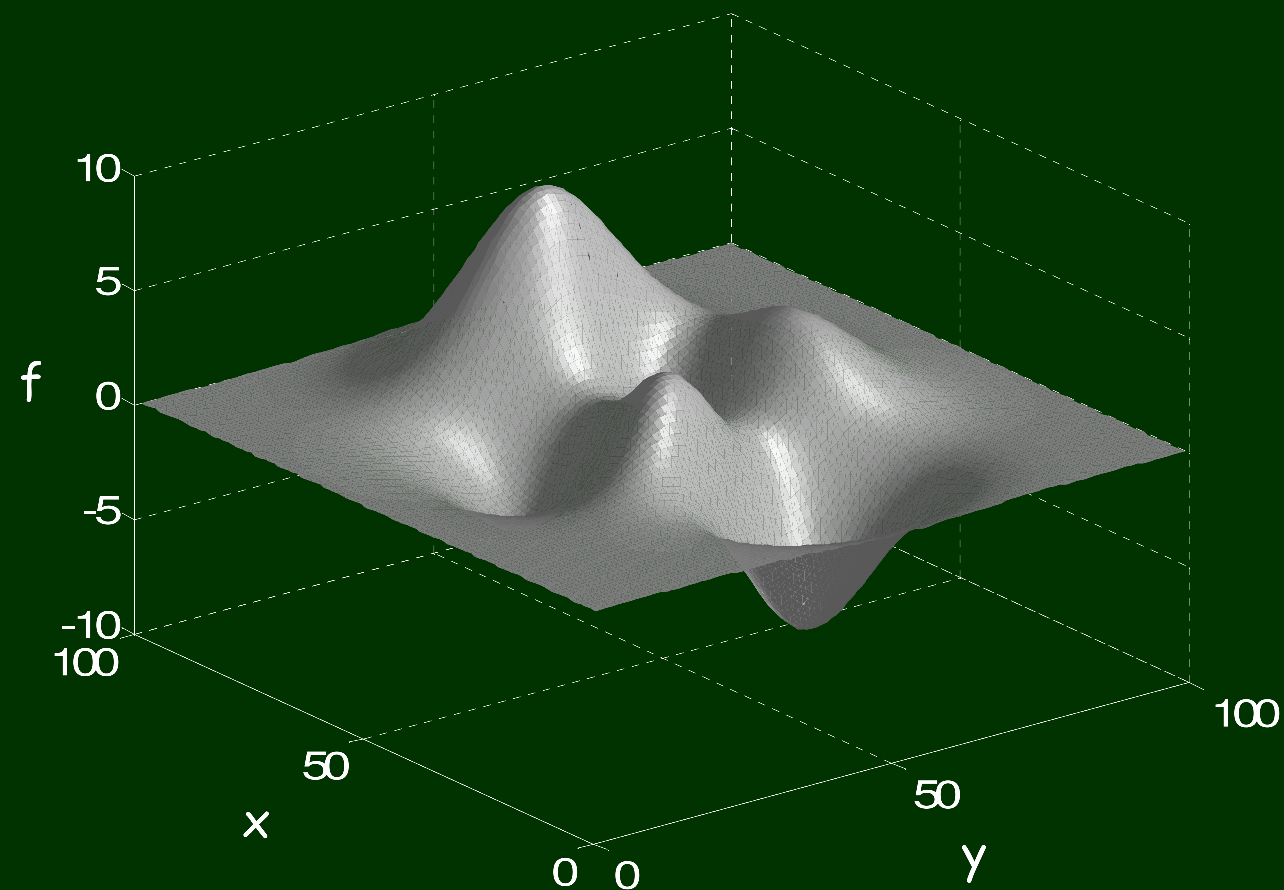
Numerički

3	4	8	9	5	0	5	4	3	8
4	5	6	8	5	4	6	8	56	4
6	4	67	5	7	5	7	9	0	6
35	3	56	78	9	0	7	6	4	34
3	5	5	76	89	0	8	76	65	5
4	66	87	9	0	9	87	6	5	4
3	83	3	34	54	5	5	56	55	5
36	8	98	9	9	76	5	54	4	56
28	39	8	7	6	5	54	4	34	3
445	56	7	8	9	00	6	87	65	54

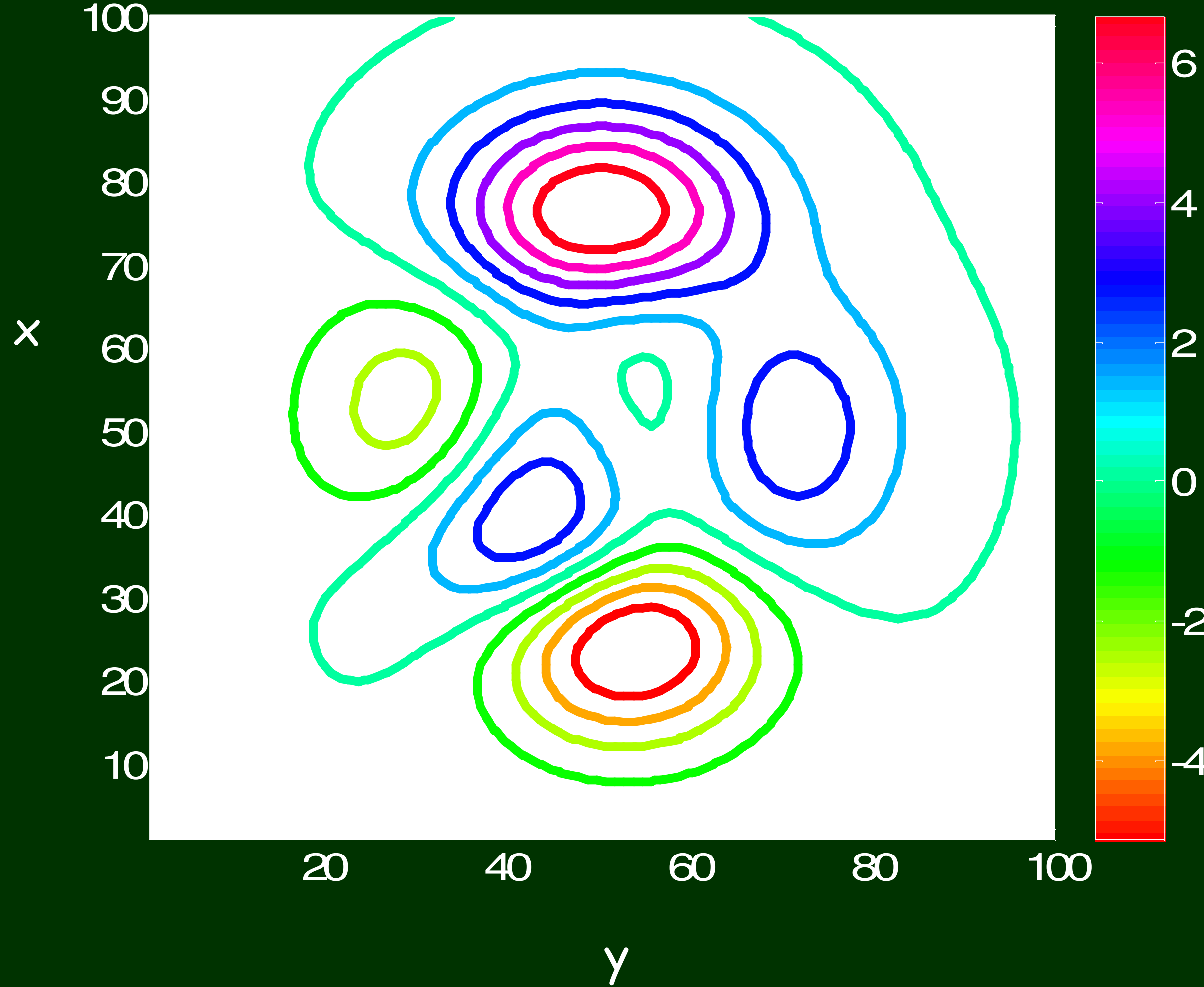
(… crno-bela slika je skalarno polje)

Grafički

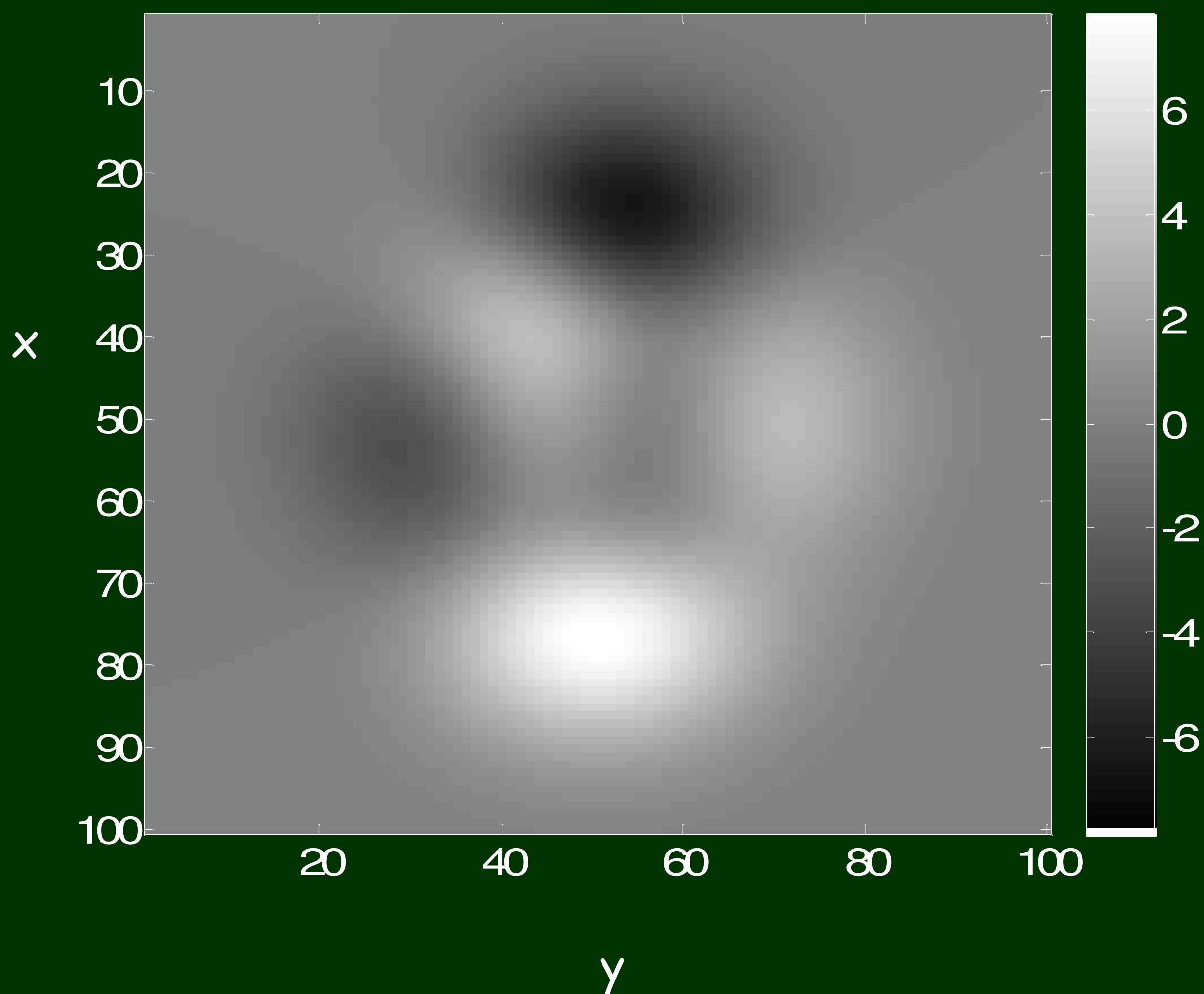
Crtež-Površ (2D skalarno polje)



Konturni crtež (isto 2D skalarno polje)



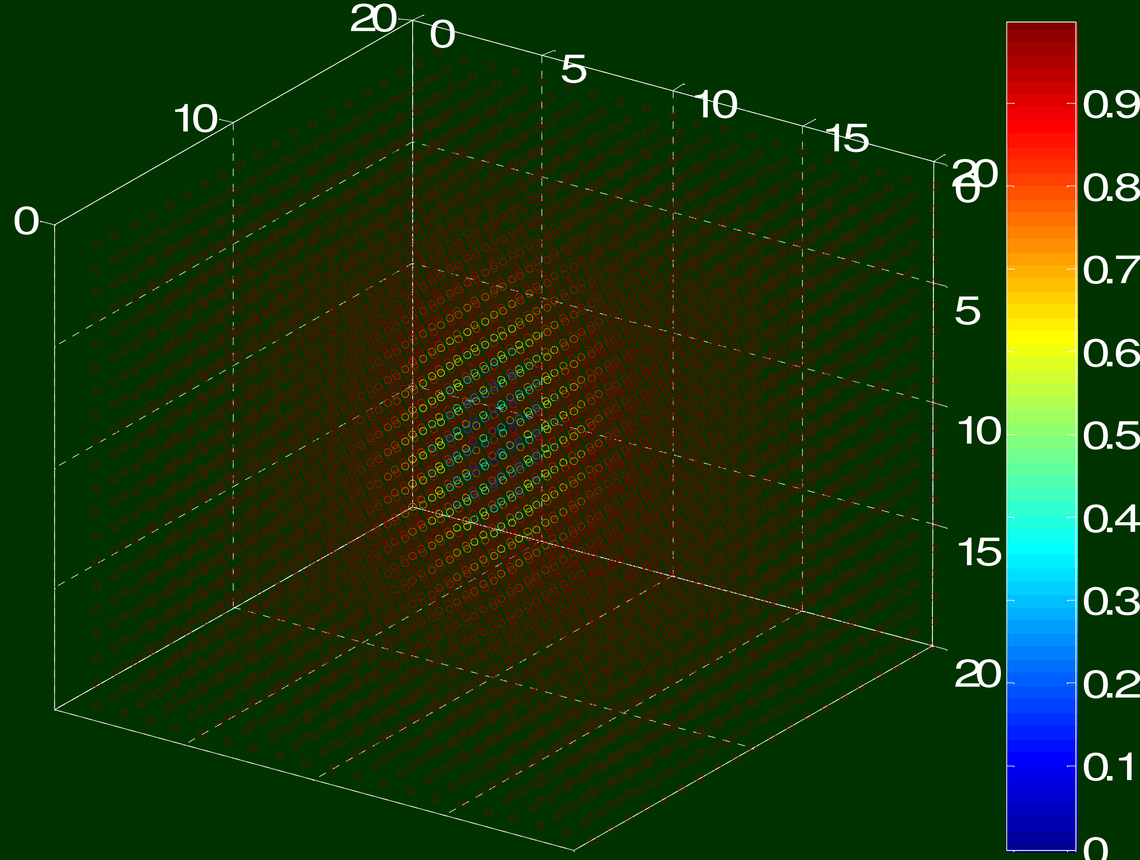
Slika (isto 2D skalarno polje)



3D skarano polje

$$S(x,y,z)=1-e^{-(x-10)^2/3}e^{-(y-10)^2/2}e^{-(z-10)^2/5}$$

3D crtež tačaka sa bojom koja daje vrednost polja:



Vektorsko polje: vektorska veličina definisana u svakoj tački 2D ili 3D prostora

Analitički

$$\vec{\mathbf{V}}(x,y,z)=V_x\hat{i}+V_y\hat{j}+V_z\hat{k}$$

Funkcije od (x,y,z)

Numerički

$$\begin{matrix}2&6&5&8\\5&2&3&3\\4&4&5&5\\1&5&0&2\end{matrix}\,\hat{i}+\begin{matrix}2&6&5&8\\5&2&3&3\\4&4&5&5\\1&5&0&2\end{matrix}\,\hat{j}+\begin{matrix}2&6&5&8\\5&2&3&3\\4&4&5&5\\1&5&0&2\end{matrix}\,\hat{k}$$

(…Slike u boji su vektorska polja)

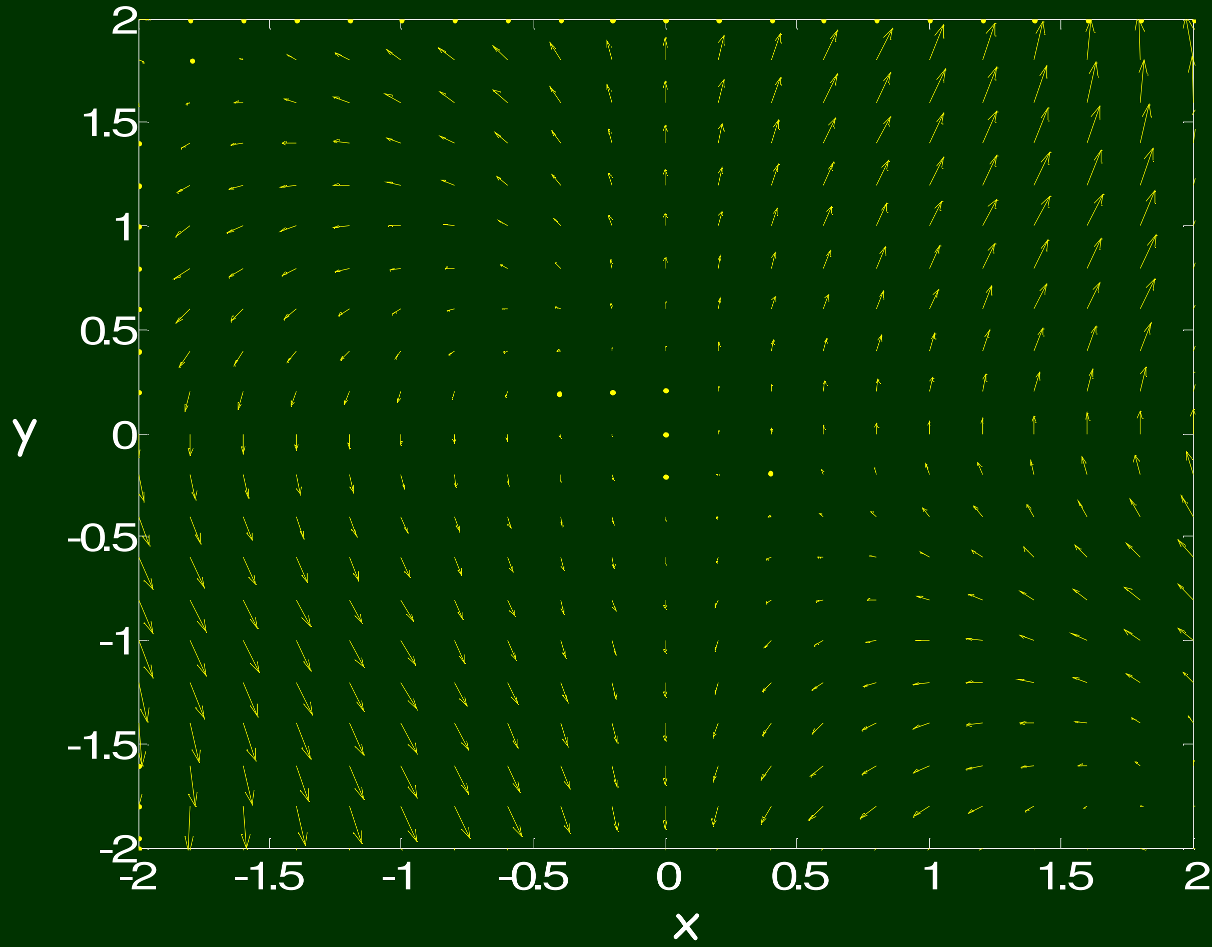
2D npr:

$$\vec{\mathbf{S}}(x,y)=\sin(xy)\hat{i}+(x+y)\hat{j}$$

Grafički

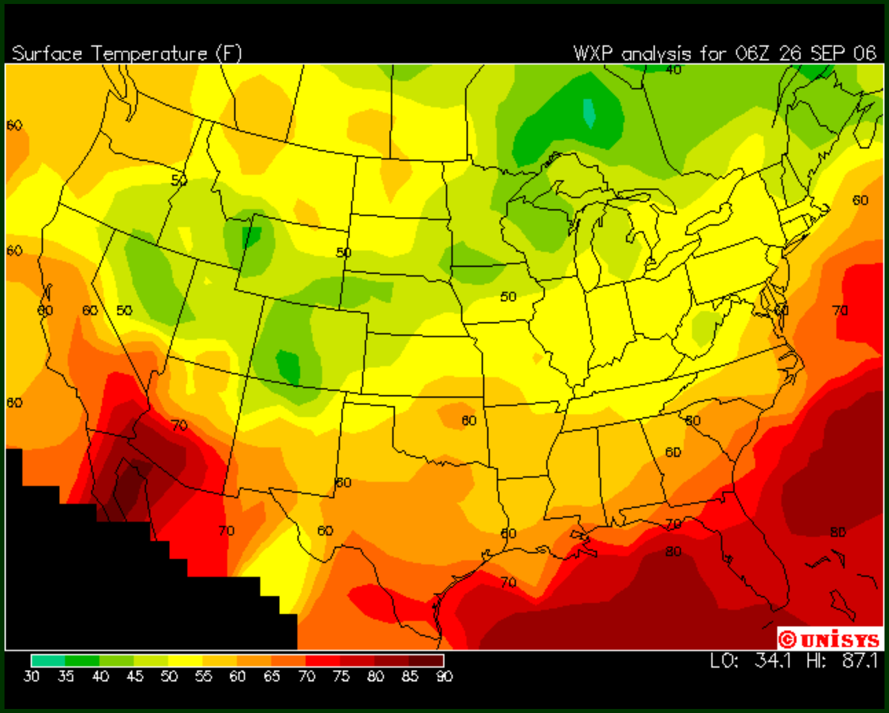
(2D vektorsko polje): dužina strelica = vrednost polja

$$\vec{\mathbf{V}}(x,y)=\sin(xy)\hat{i}+(x+y)\hat{j}$$

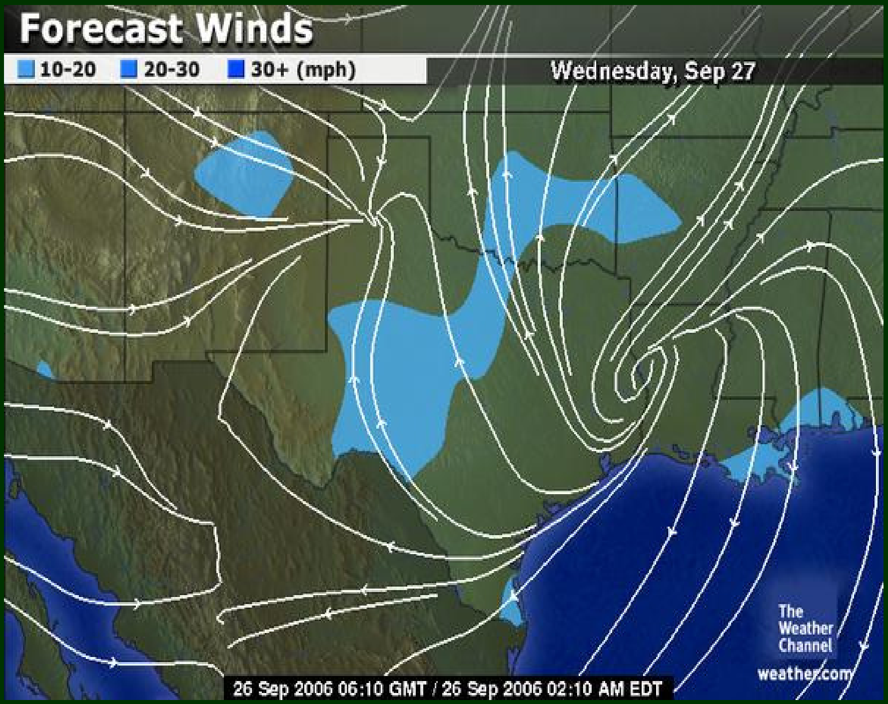


Dva polja

Temperaturska mapa:
skalarno polje



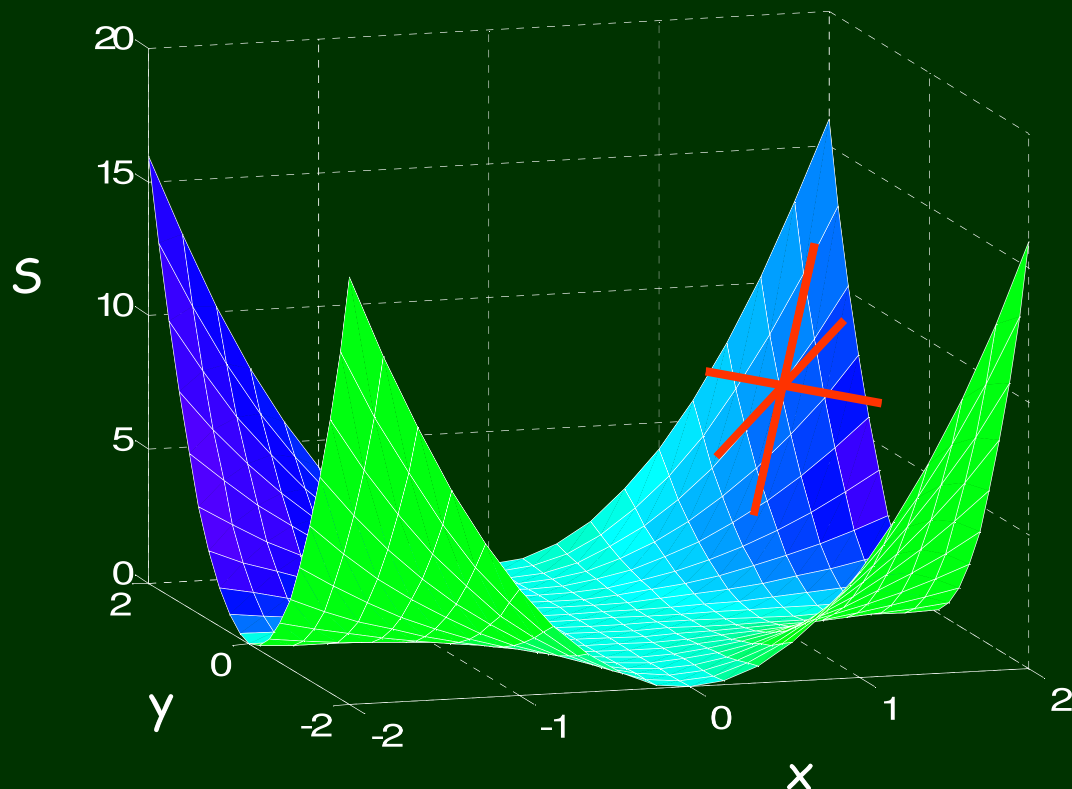
Mapa vetrova:
Vektorsko polje



Gradijent

"izvod polja"

$$S = x^2 y^2$$



Izvod (nagib) zavisi od pravca!

Totalni Diferencijal: $dS = \left(\frac{\partial S}{\partial x}\right)dx + \left(\frac{\partial S}{\partial y}\right)dy$

Sličan je skalarnom proizvodu: $dS = \left(\frac{\partial S}{\partial x}\hat{i} + \frac{\partial S}{\partial y}\hat{j}\right)\cdot\left(dx\hat{i} + dy\hat{j}\right)$

$$dS = (\nabla S)\cdot\left(d\vec{l}\right)$$

$$\boxed{\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}}$$

"delta"

"nabla"

Delta nije vektor i ne množi polje – delta je operator!

Premda, delta deluje kao vektor:

1. Deluje na skalarno polje: gradijent ∇S

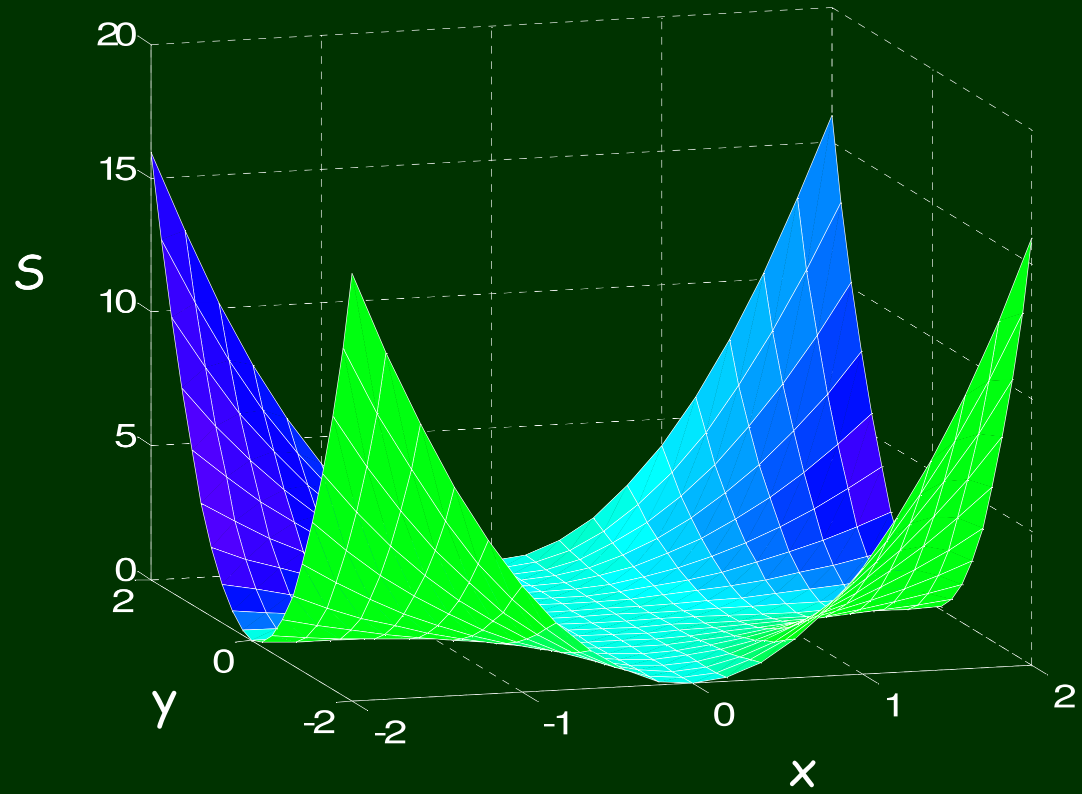
2. Kao skalarni proizvod sa vektorskim poljem: divergence $\nabla \cdot \vec{\mathbf{V}}$

3. Kao vektorski proizvod sa vektorsim poljem: rotor $\nabla \times \vec{\mathbf{V}}$

Primer gradijenta:

$$S = x^2 y^2$$

$$\nabla S = 2xy^2\hat{i} + 2yx^2\hat{j}$$



At (1,1)

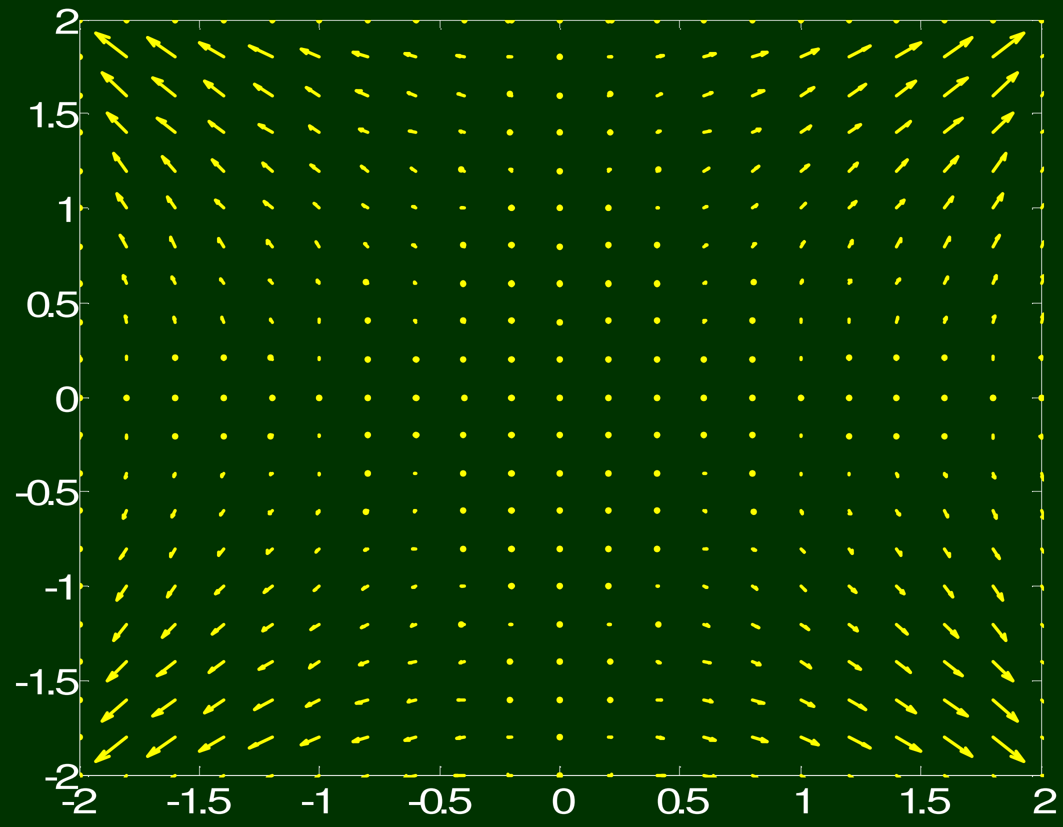
$$\nabla S = 2\hat{i} + 2\hat{j}$$

At (-1,1)

$$\nabla S = -2\hat{i} + 2\hat{j}$$

Gradijent skalarnog polja je vektorsko polje čije komponente su izvodi skalarnog polja duž svake ose:

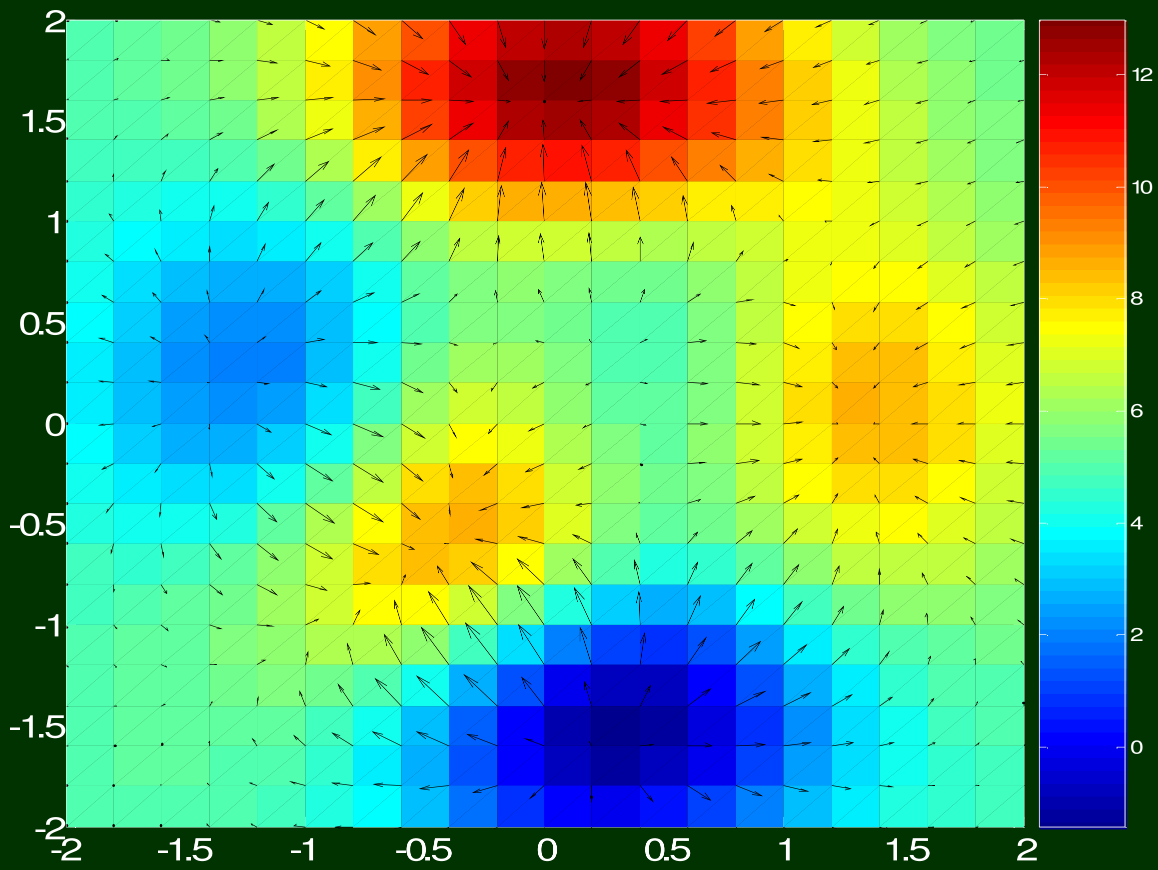
$$\nabla S = 2xy^2\hat{i} + 2yx^2\hat{j}$$



U svakoj tački vektor gradijenta je usmeren duž pravca maksimalnog nagiba i njegova vrednost je jednaka nagibu u tom pravcu.

$$\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$$

∇S = gradijent



Kolor crtež = skalarno polje strelice = gradijent

Fundamentalne Teoreme

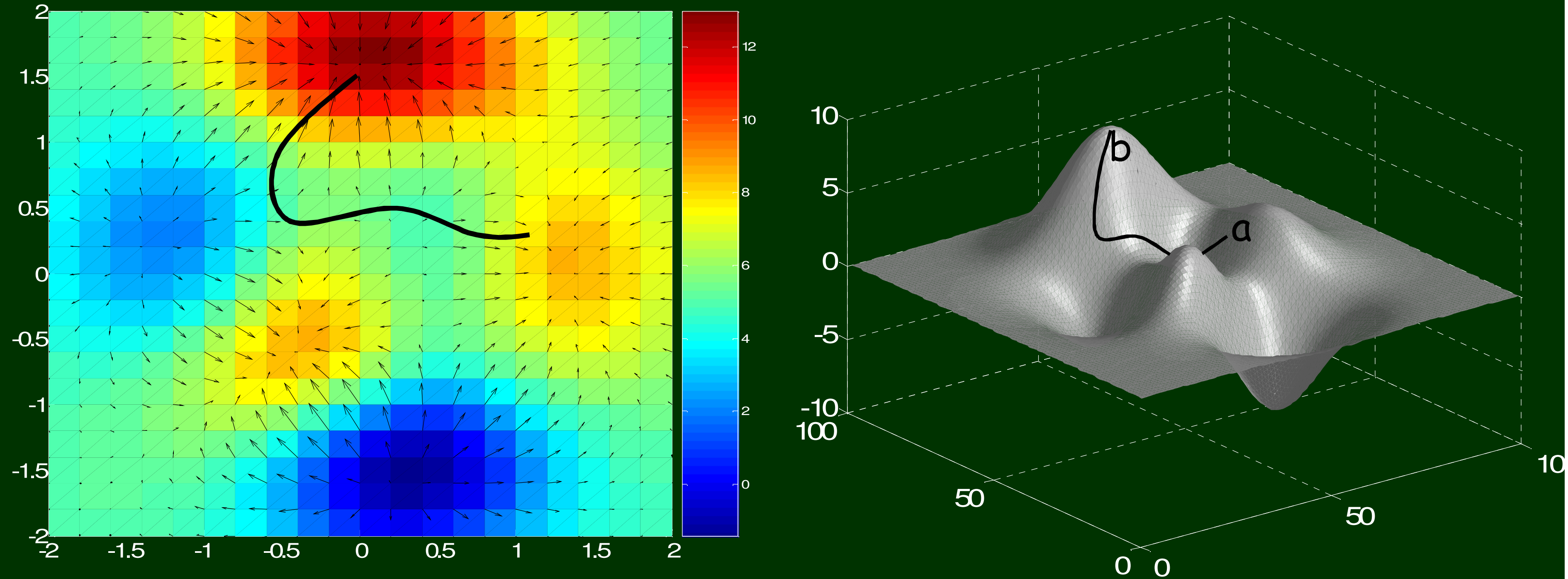
$$\int_a^b \frac{df}{dx}dx = f(b) - f(a)$$

Sa poljima , delta operator nam daje tri fundamentalne teoreme:

1. Fundamentalna Teorema gradijenta

$$\int_{\mathbf{a}}^{\mathbf{b}} (\nabla S)\cdot d\vec{l} = S(b) - S(a)$$

(Integral od izvoda u nekom opsegu = vrednost na granici)



2
1.5
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-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
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1
1.5
2
12
10
8
6
4
2
0
10
5
0
-5
-10
100
50
0
0
50
10
b
a

$$\nabla \cdot \vec{\mathbf{V}}$$

Divergencija vektorskog polja

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot \left(V_x \hat{i} + V_y \hat{j} + V_z \hat{k} \right)$$

$$\frac{\partial}{\partial x}V_x + \frac{\partial}{\partial y}V_y + \frac{\partial}{\partial z}V_z$$ (skalarno polje)

$$\vec{\mathbf{V}}(x, y) = \cos(xy)\hat{i} + \left(x^2 + y^2\right)\hat{j}$$

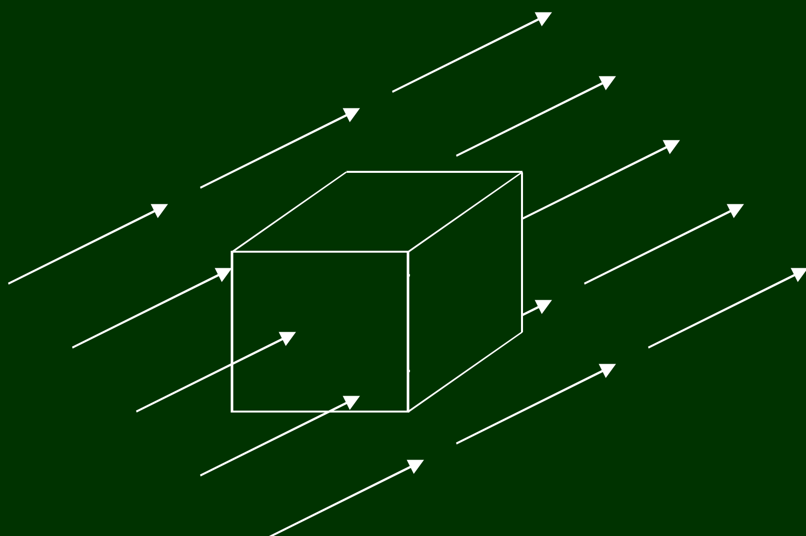
$$\nabla \cdot \vec{\mathbf{V}} = -y\sin(xy) + 2y$$

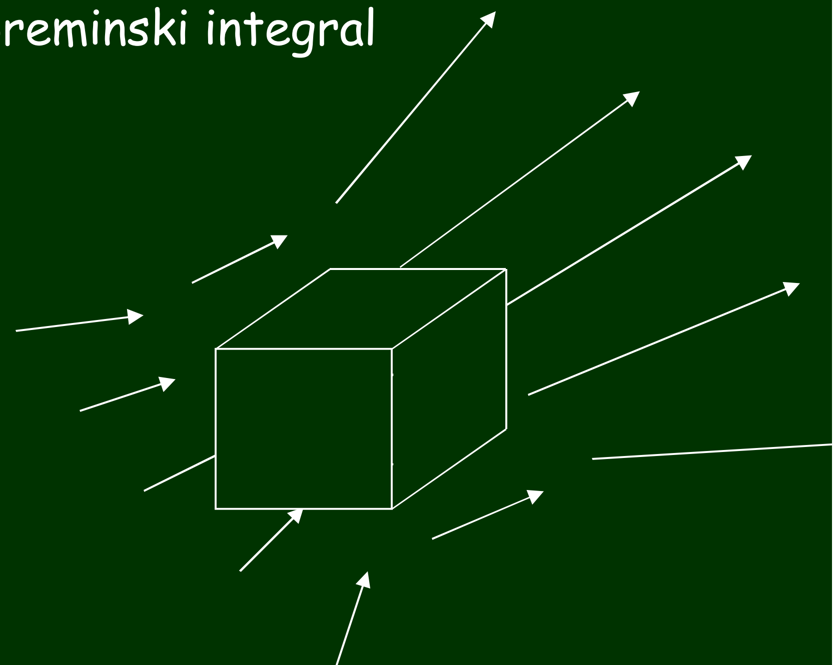
2. Fundamentalna teorema divergencije

(Gaus-Ostrogradski)

$$\oiint_S \vec{F} \cdot \vec{dS} = \iiint_V \nabla \cdot \vec{F} dV$$

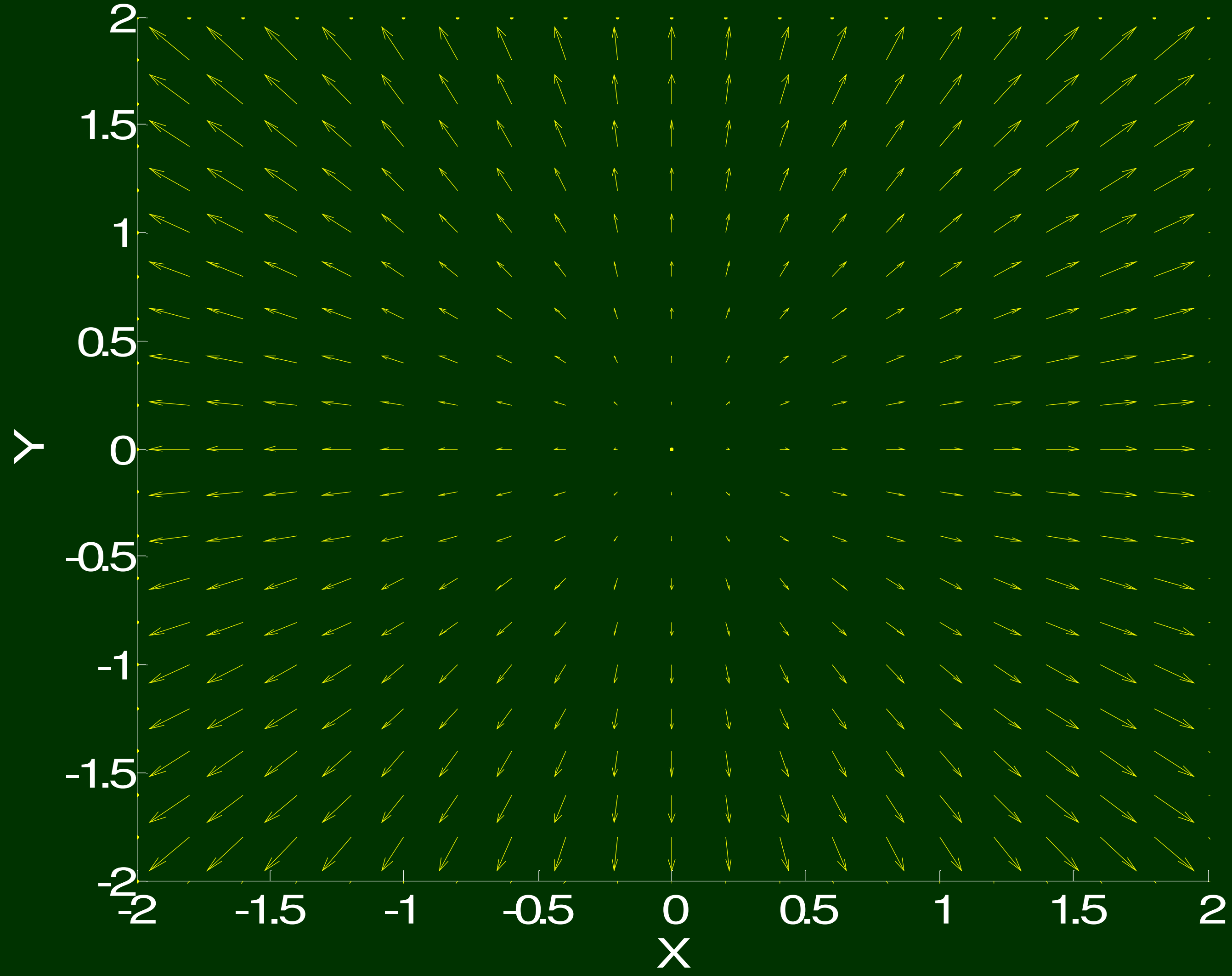
površinski integral zapreminski integral

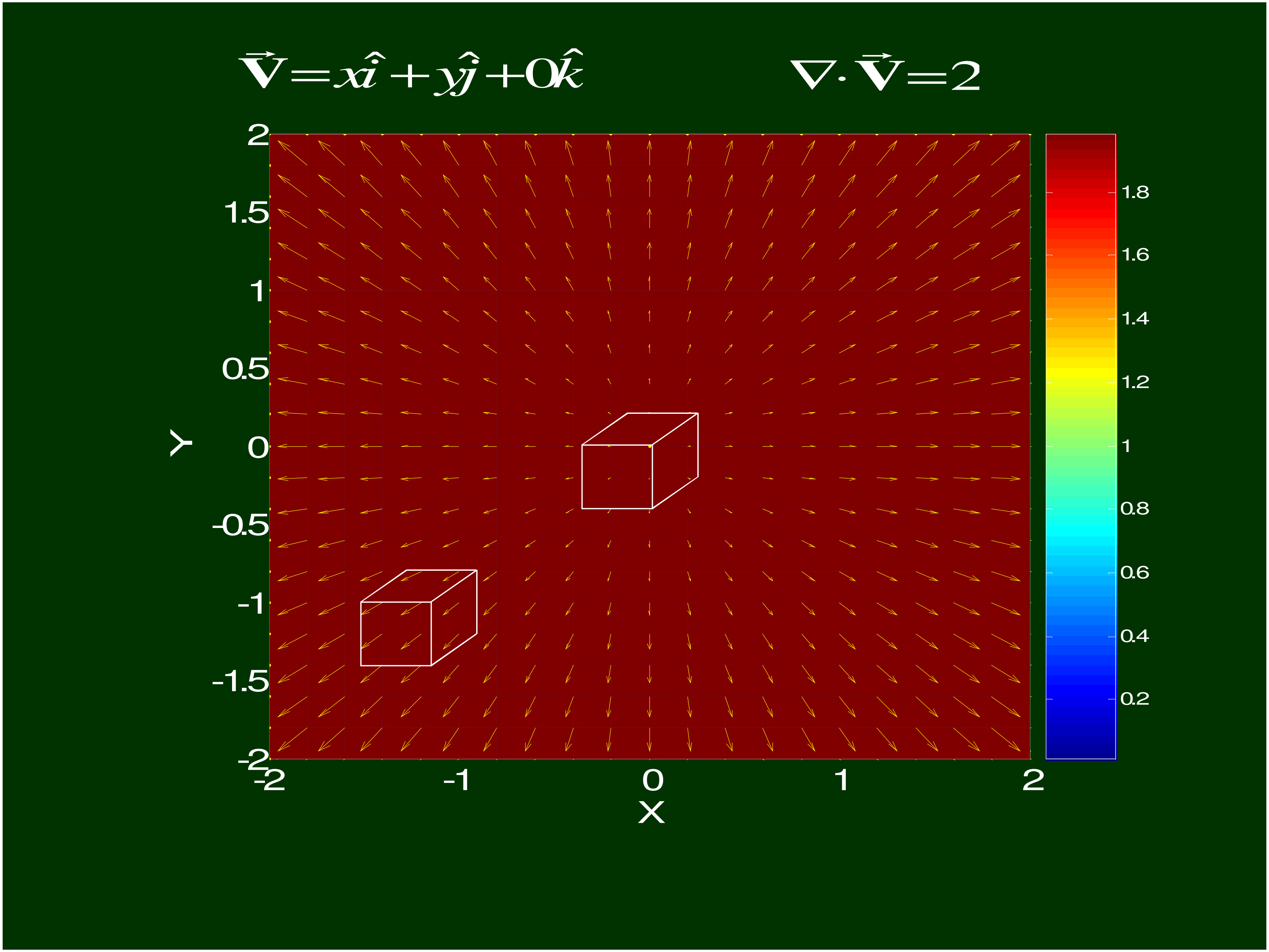




(Integral od izvoda u nekom opsegu = vrednosti na granici)

$$\vec{\mathbf{V}} = x\hat{i} + y\hat{j} + 0\hat{k}$$




$\vec{\mathbf{V}} = x\hat{i} + y\hat{j} + 0\hat{k}$
$\nabla \cdot \vec{\mathbf{V}} = 2$
Y
X
2
1.5
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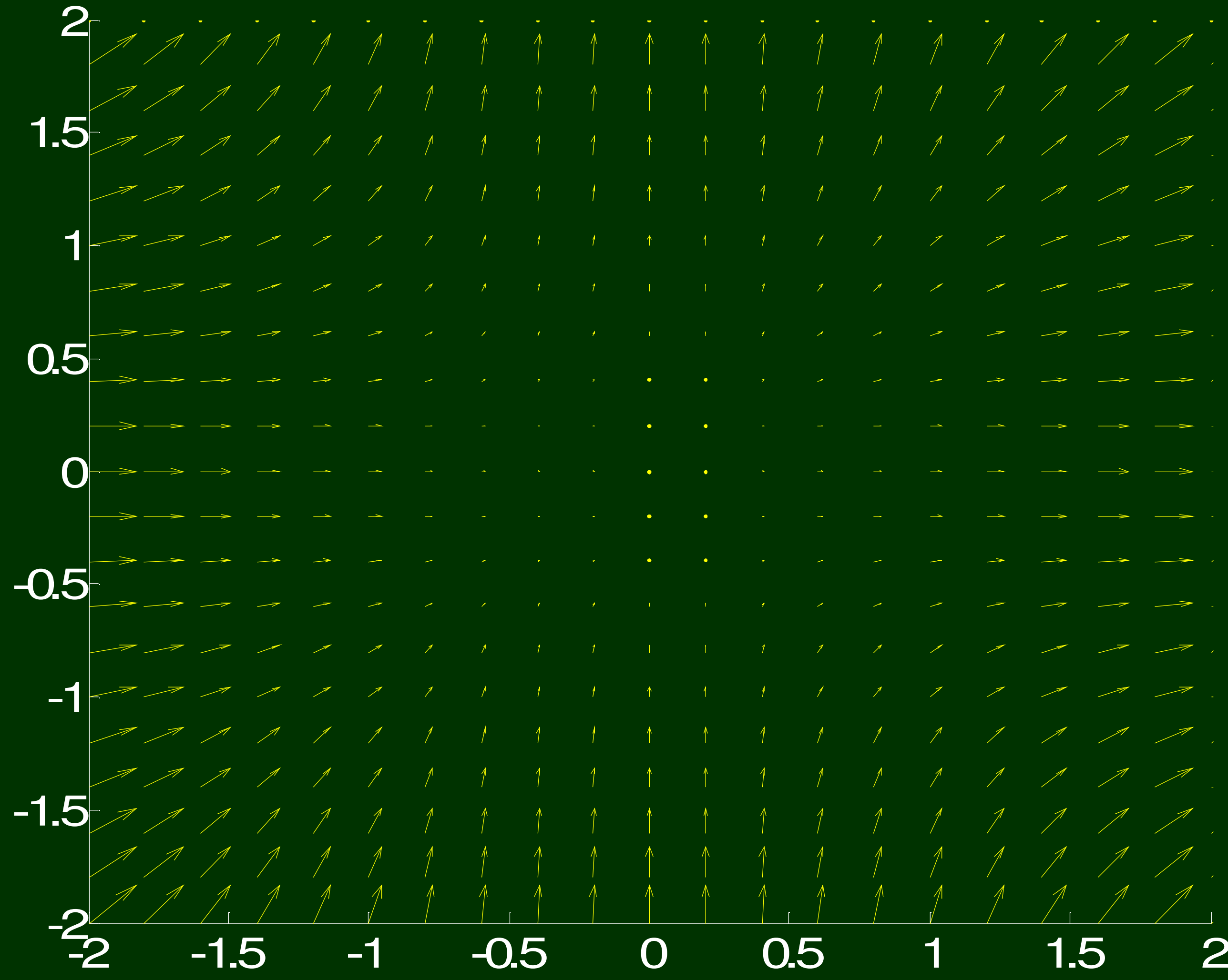
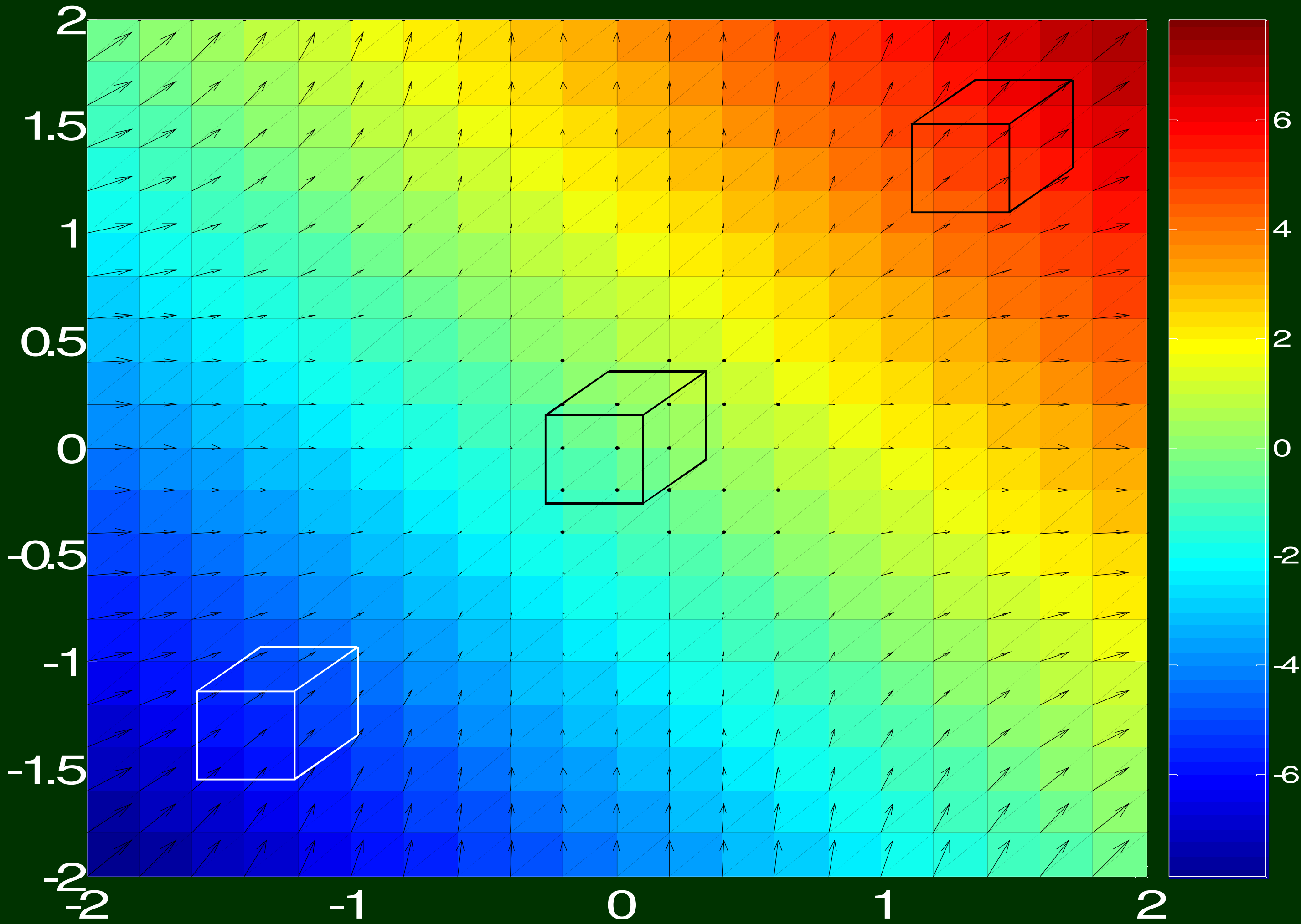
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2

$$\vec{\mathbf{V}} = x^2\hat{i} + y^2\hat{j}$$



$$\vec{\mathbf{V}} = x^2\hat{i} + y^2\hat{j} \qquad \nabla\cdot\vec{\mathbf{V}} = 2x + 2y$$



$$\nabla \times \vec{\mathbf{V}}$$

Rotor vektorskog polja

$$\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\times\left(V_x\hat{i}+V_y\hat{j}+V_z\hat{k}\right)$$

$$\det\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ V_x & V_y & V_z\end{bmatrix}$$

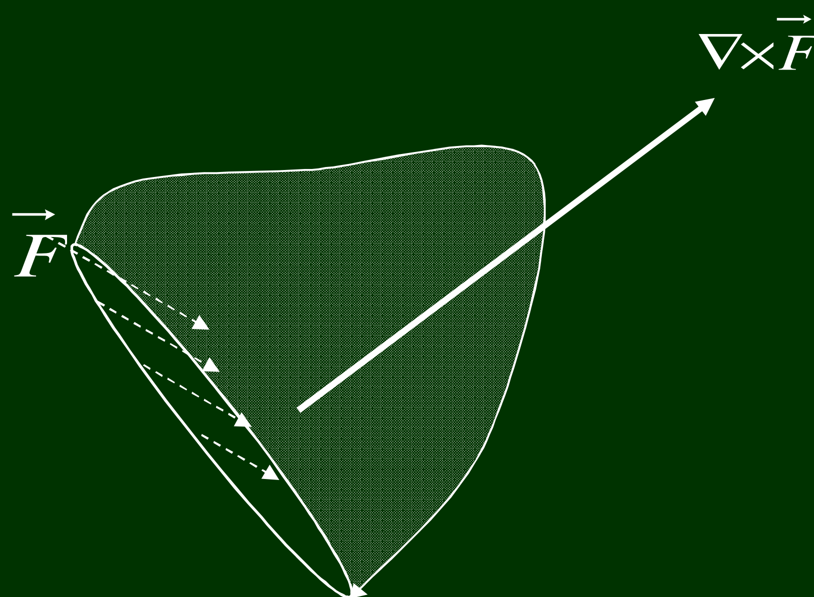
"Koliko vektorsko polje može nešto da uvrne"

3. Fundamentalna teorema za rotor

(Stoksova teorema)

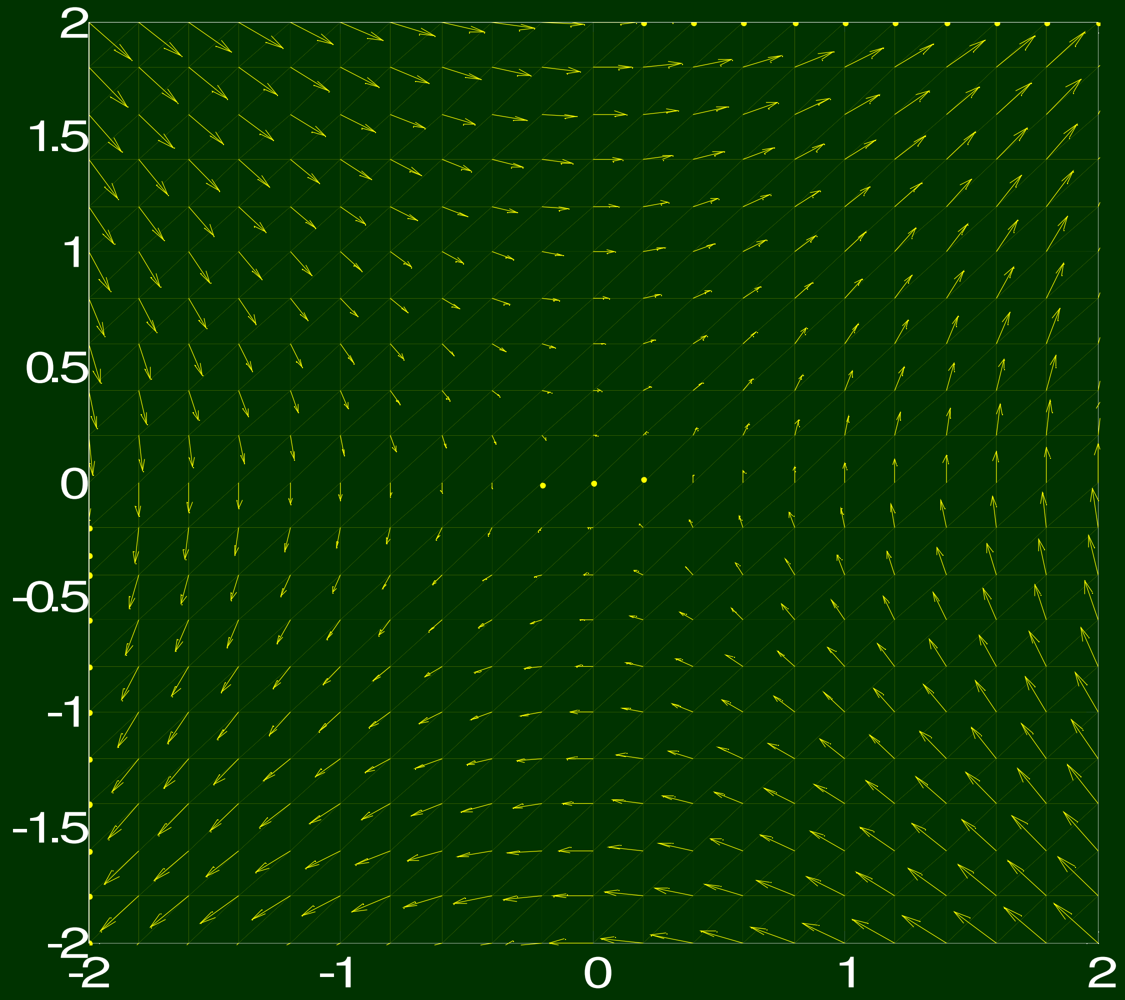
$$\oint_L \vec{F}\cdot\vec{dl} = \iint_S \nabla\times\vec{F}\cdot d\vec{S}$$

zatvoreni linijski integral   Otvoreni površinski integral

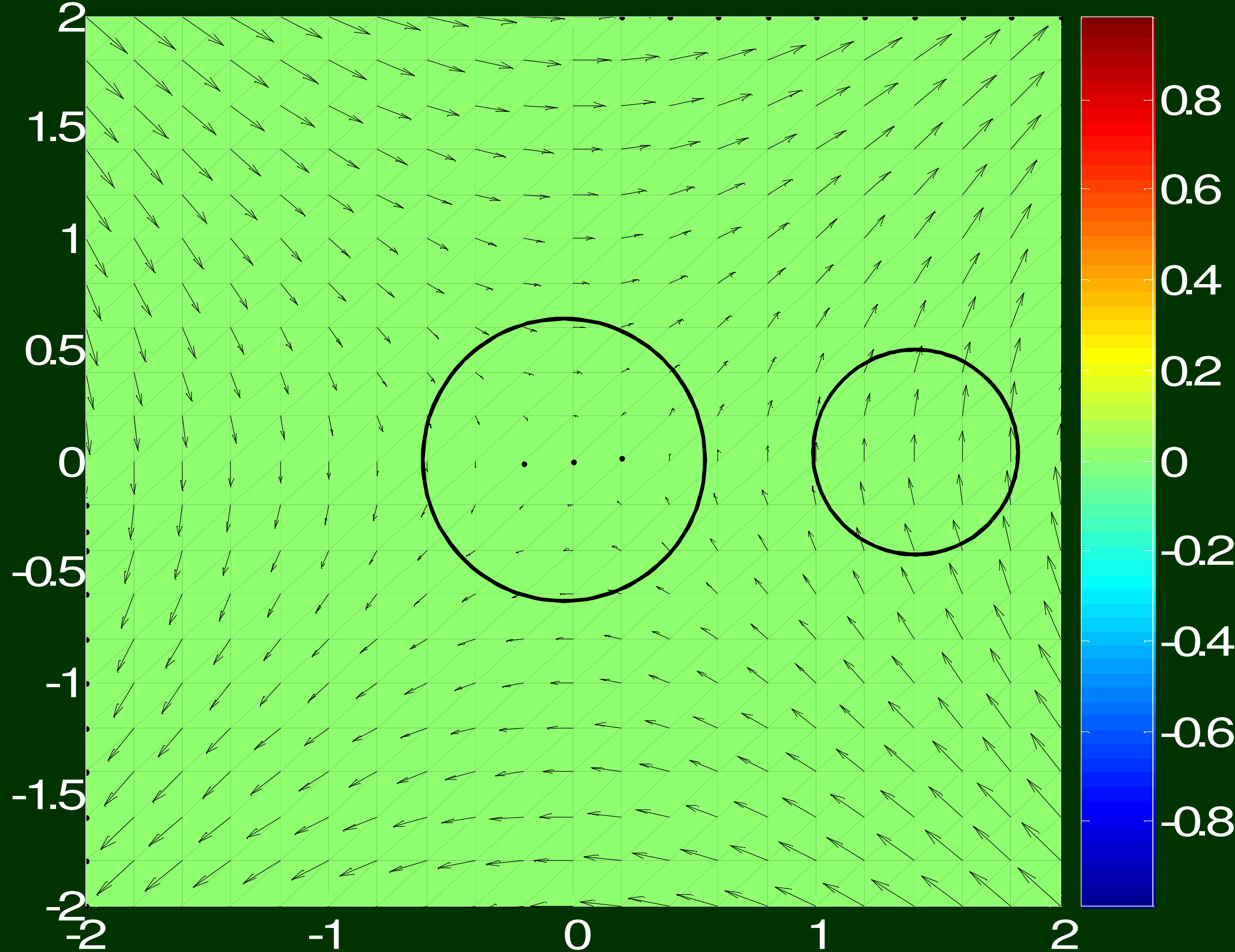


(Integral od izvoda u nekom opsegu = vrednosti na granici)

$$\vec{\mathbf{V}} = x\hat{i} + y\hat{j} + 0\hat{k}$$

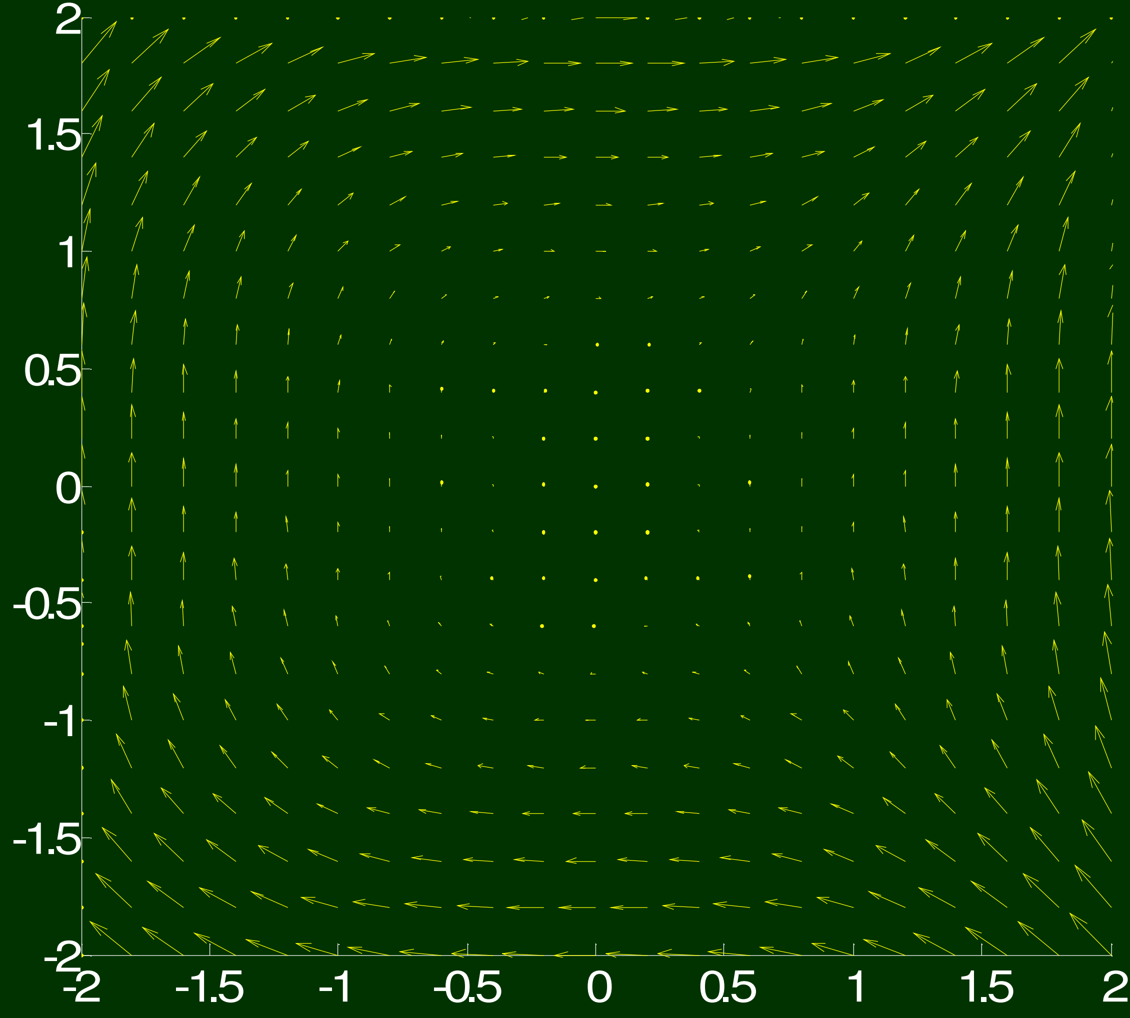


$$\vec{\mathbf{V}} = x\hat{i} + y\hat{j} + 0\hat{k} \qquad \nabla \times \vec{\mathbf{V}} = 1 - 1 = 0$$



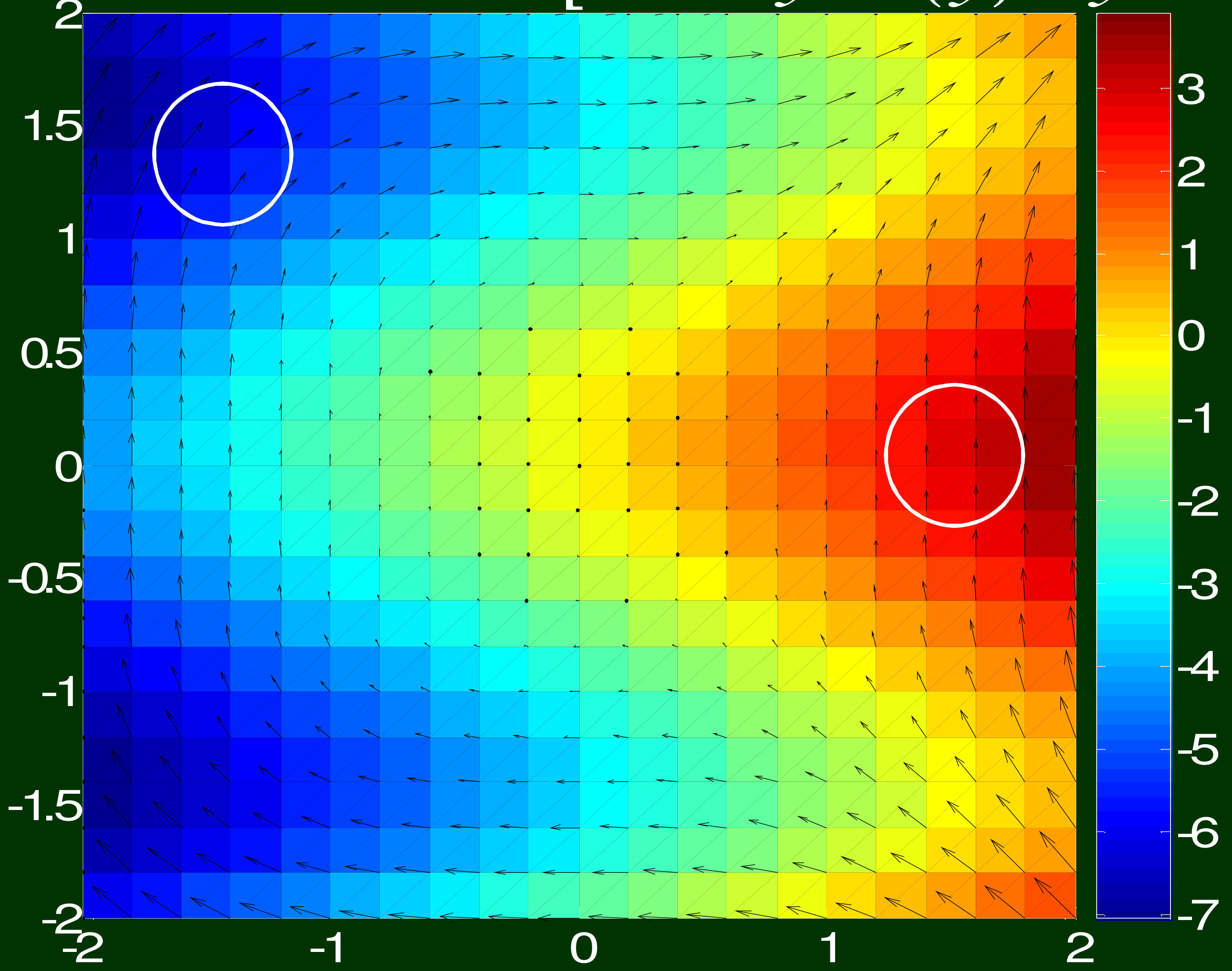
Kolor crtež = z komponenta rotora(V)

$$\vec{\mathbf{V}} = y^2 \sin(y)\hat{i} + x^2 \hat{j} + 0\hat{k}$$



$$\vec{\mathbf{V}} = y^2 \sin(y)\hat{i} + x^2\hat{j} + 0\hat{k}$$

$$\nabla \times \vec{\mathbf{V}} = \left[2x - 2y\sin(y) + y^2\cos(y)\right]\hat{k}$$



Kolor crtež = z komponenta od rotora(V)

2 polja i 3 Operacije

$S(x, y, z)$ Skalarno polje

$\vec{\mathbf{V}}(x, y, z)$ Vektorsko polje

∇S Gradijent: usmereno ka većoj vrednosti

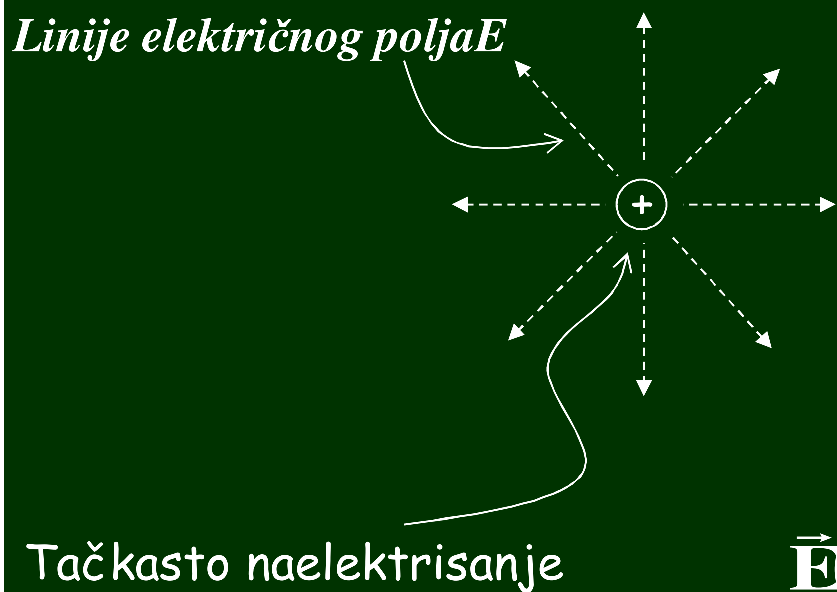
$\nabla \cdot \vec{\mathbf{V}}$ Divergencija: vectori izviru ili uviru u zapreminski element

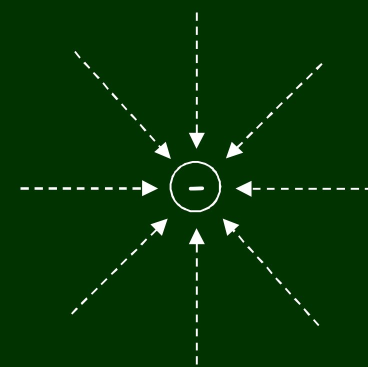
$\nabla \times \vec{\mathbf{V}}$ rotor: vectori izazivaju rotaciju površi

MAKSVELOVE JEDNAČINE

I.II. Gausov zakon

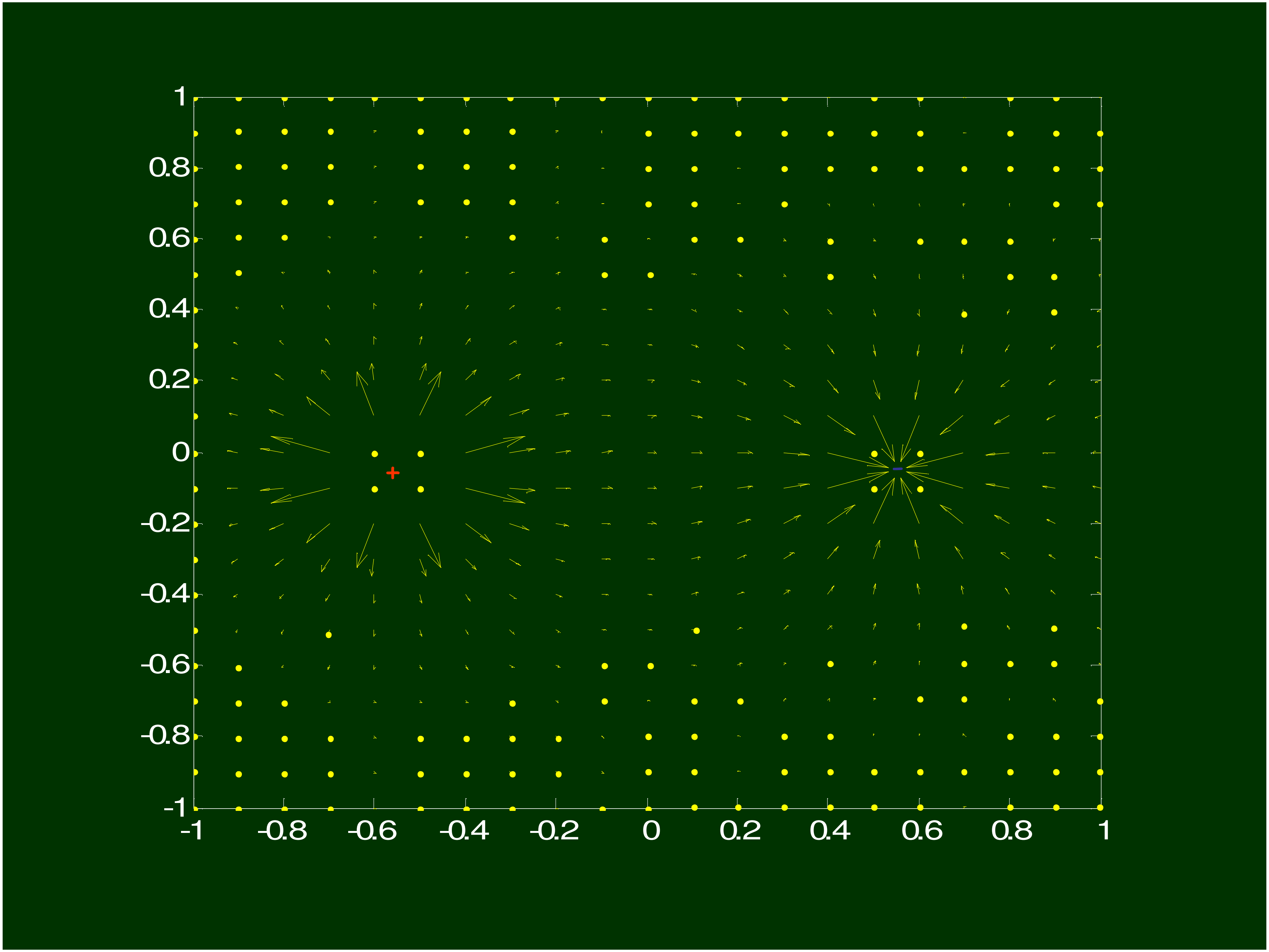
Daje vezu između raspodele naelektrisanja i električnog polja





$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

…dobro opisuje za tačkasto naelektrisanje.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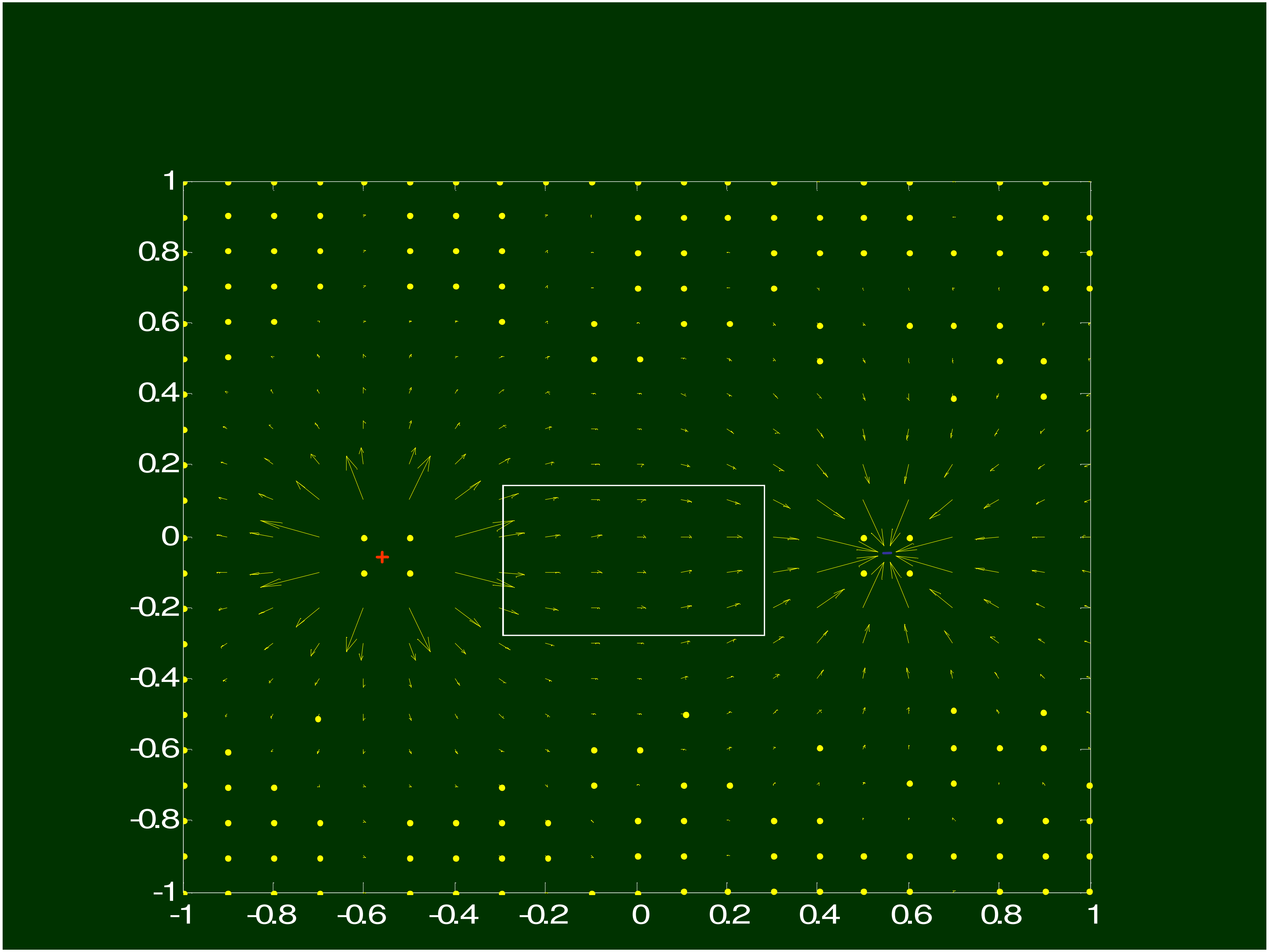

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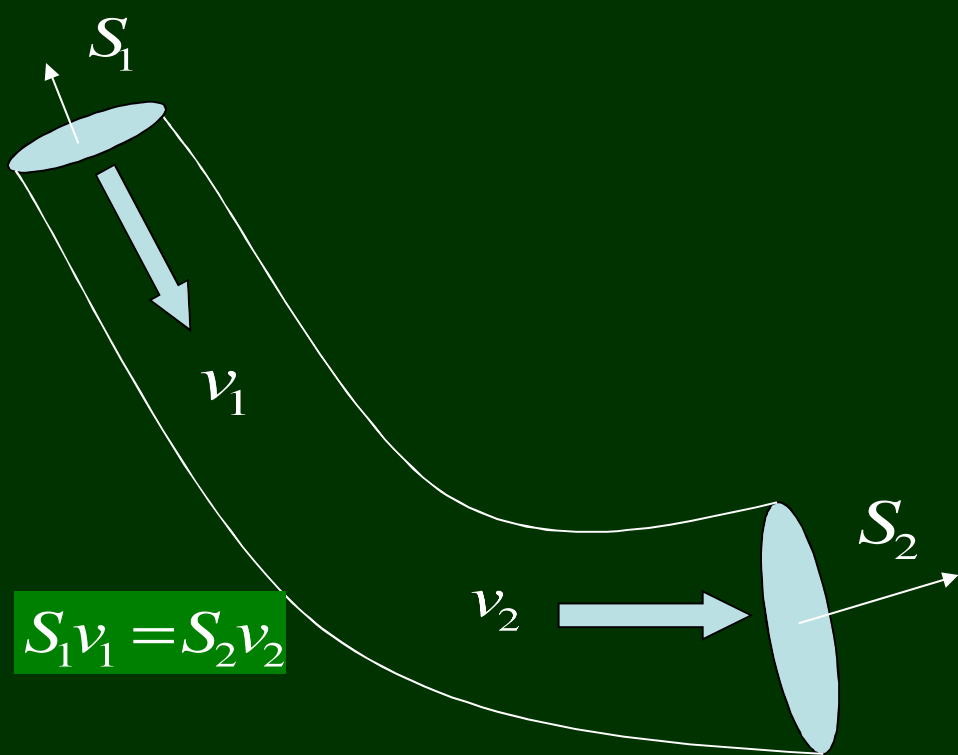
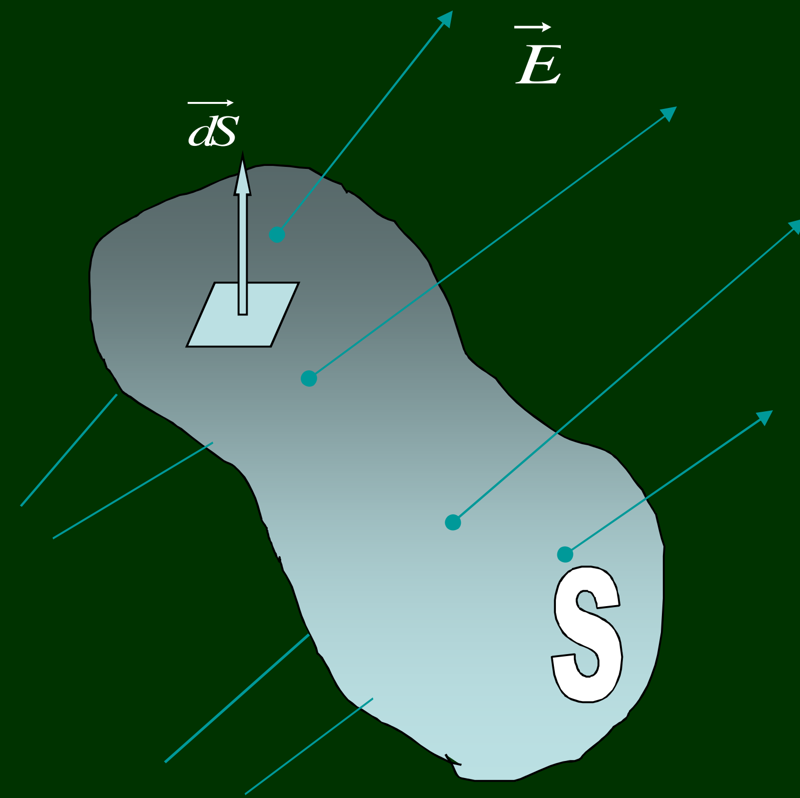
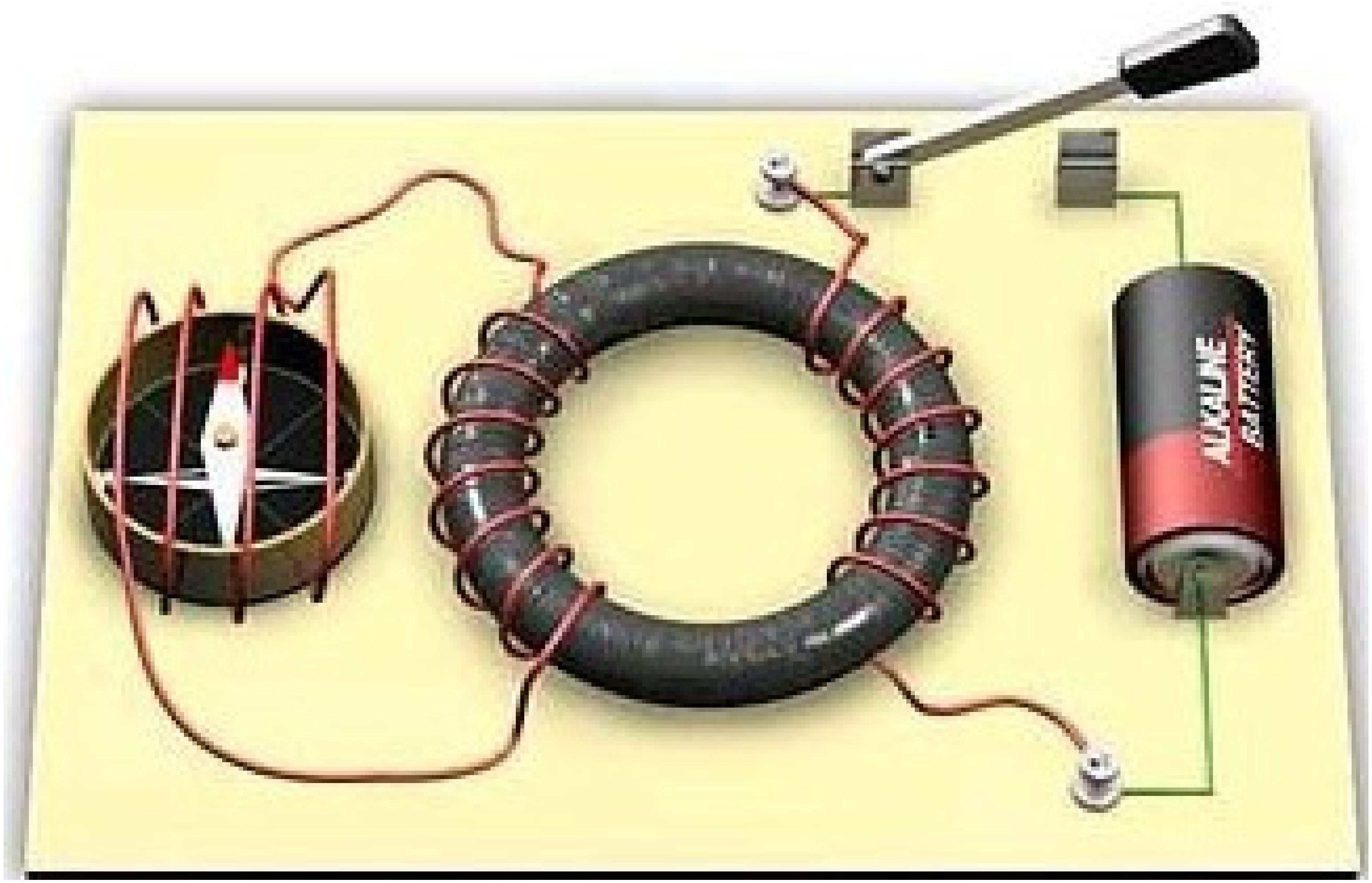




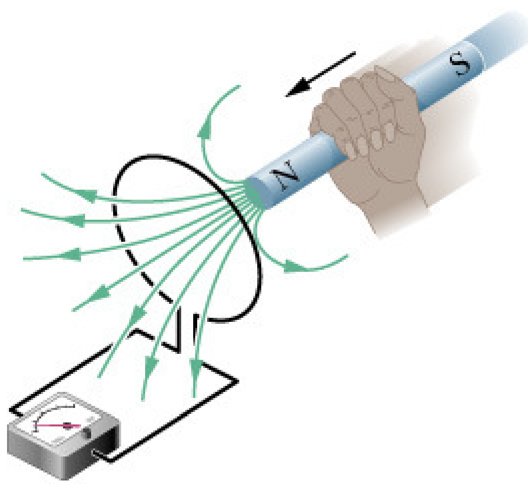
$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \iiint_V \rho \cdot dV$$

III. Faradejev zakon indukcije

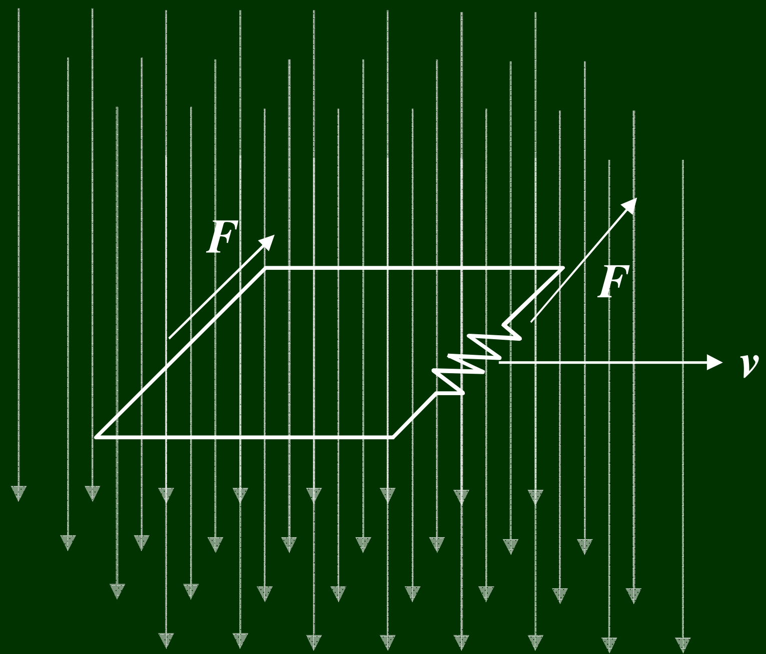


- **Promenljivi magnetni fluks generiše elektromotornu silu ems**
- **Ili, promenljivo B-polje generiše E-polje**
- **Promena magnetnog fluksa je potrebna**



$$\varepsilon = -\frac{d\Phi}{dt}$$

Sila koja deluje na provodne elektrone:

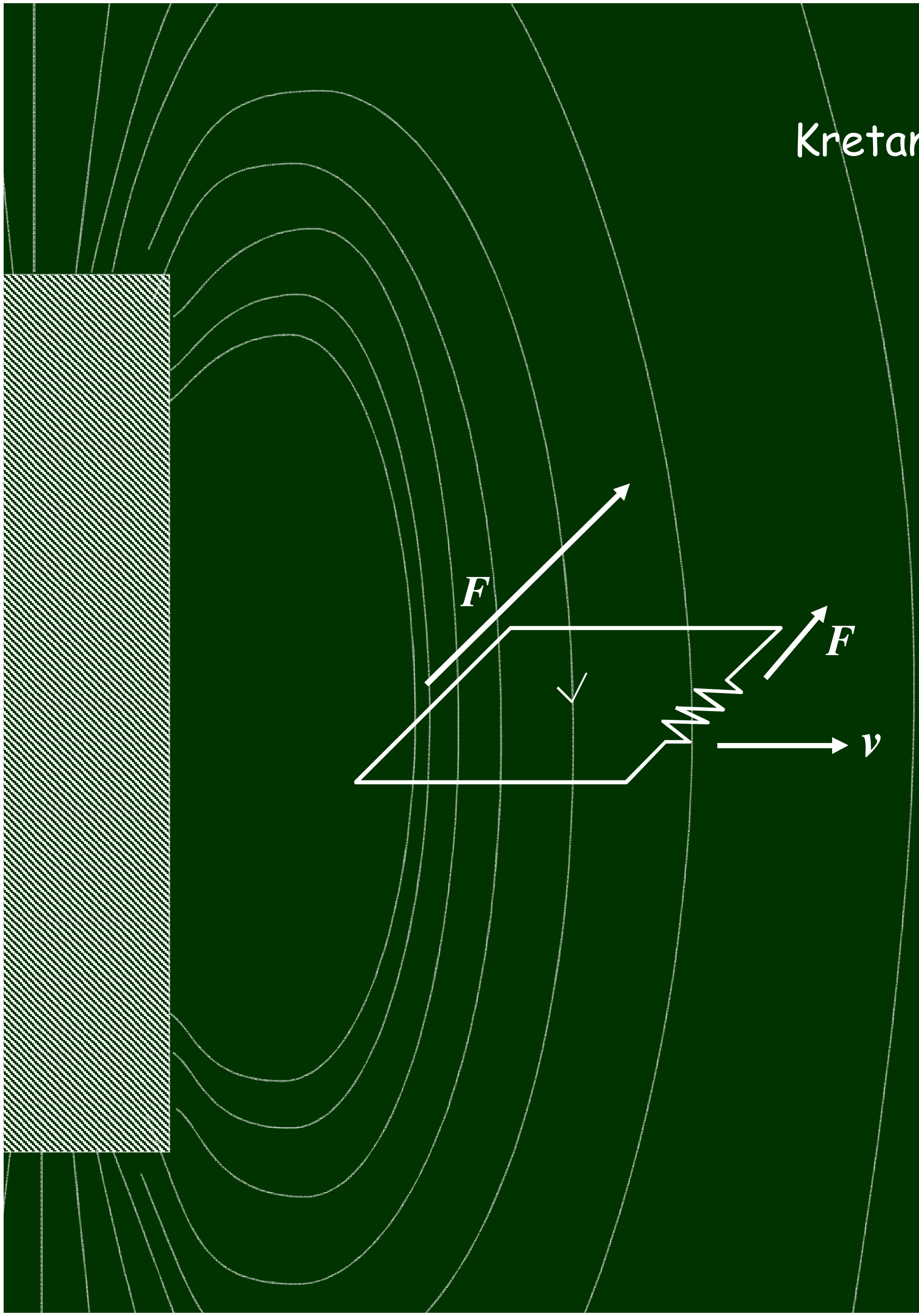


Faraday

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

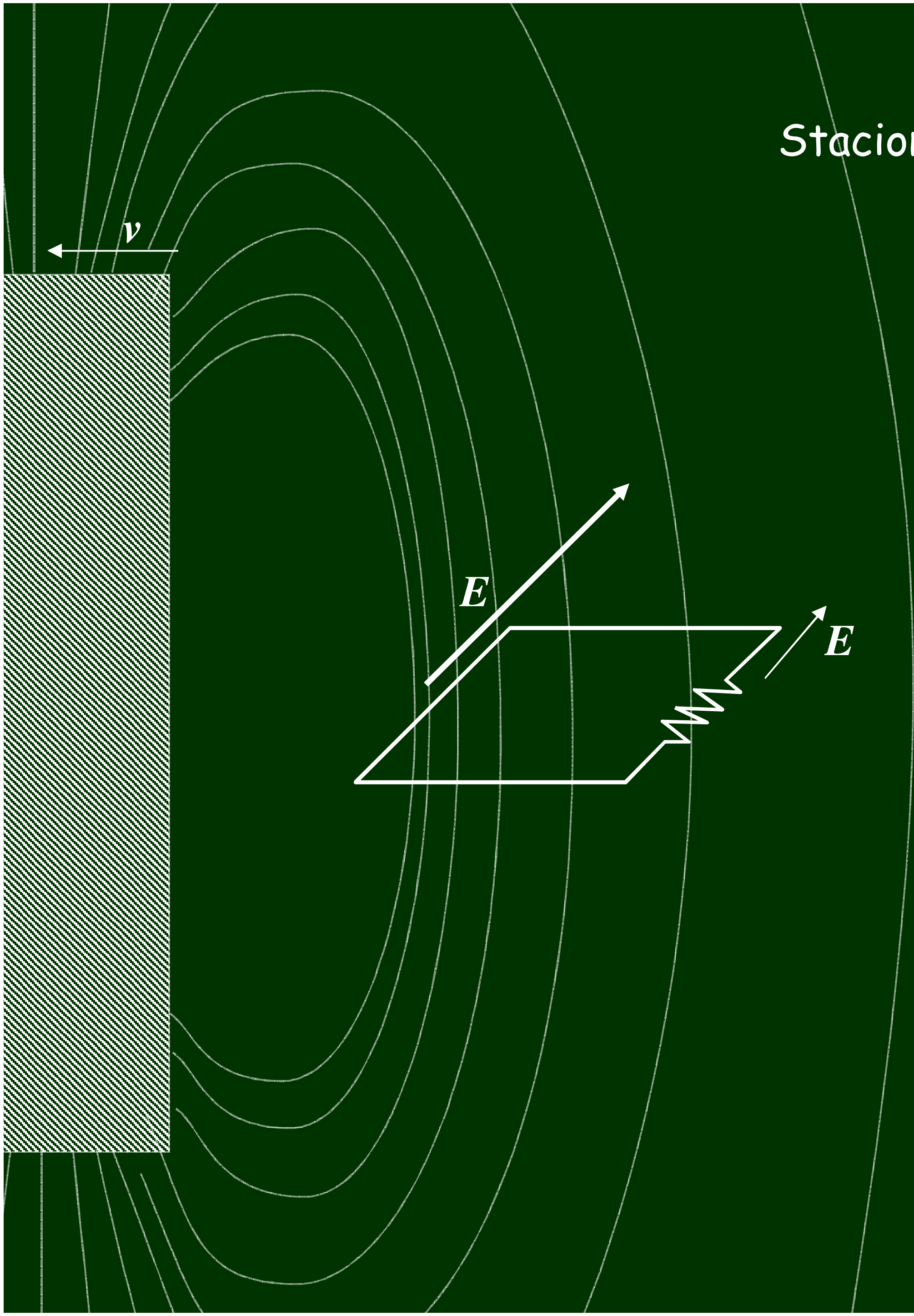
Nestankom sile: $ems = 0$

Kretanje namotaja u promenljivom B polju



$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Sila se ne poništava: $ems \neq 0$



Stacionarni kalem sa pokretnim B izvorom:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

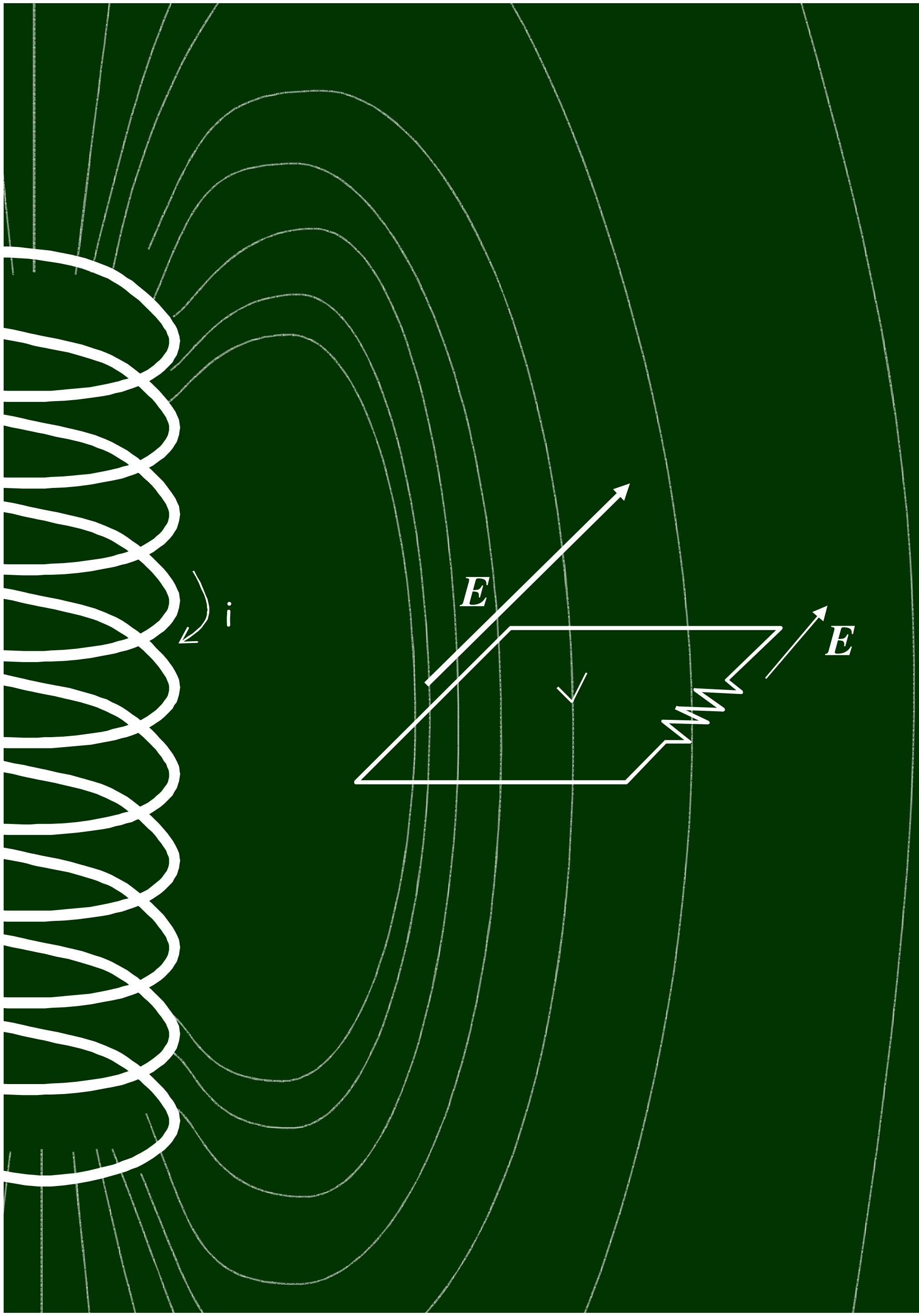
$$\vec{v} = 0$$

Ali mi još imamo ems …

Ostaje samo:

$$\vec{F} = q\vec{E}$$

Električno polje mora biti kreirano!



Stacionarni kalem i izvor ***B*** polja, ali se polje ***B*** povećava:

$$ems \neq 0$$

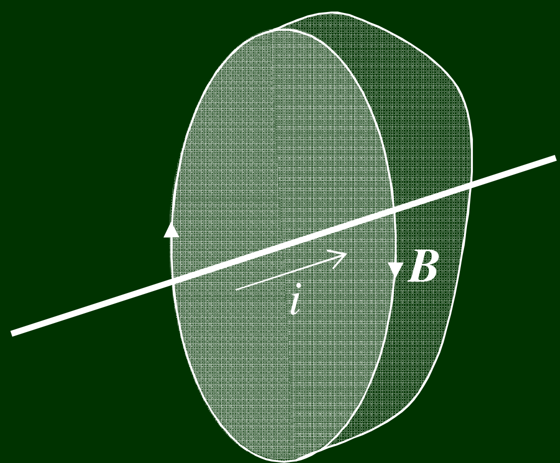
Uopšteno:

$$ems = -\frac{\partial \Phi_B}{\partial t}$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Faradejev zakon
(integralna forma)

IV. Amperov zakon



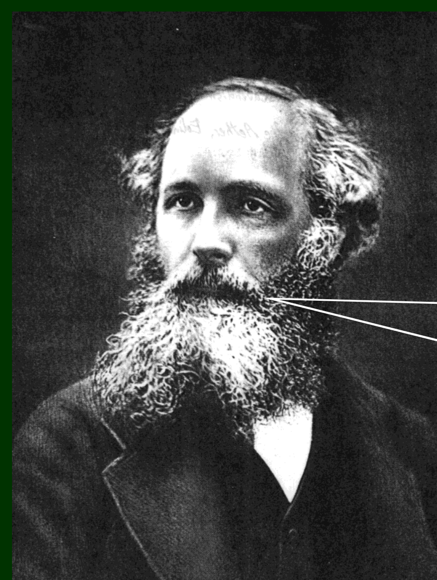
$$\oint_C \vec{\mathbf{B}} \cdot d\vec{l} = \mu_0 i_{zatvorena}$$

Oopštenije:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \iint_S \vec{J} \cdot d\vec{S}$$

J = gustina struje

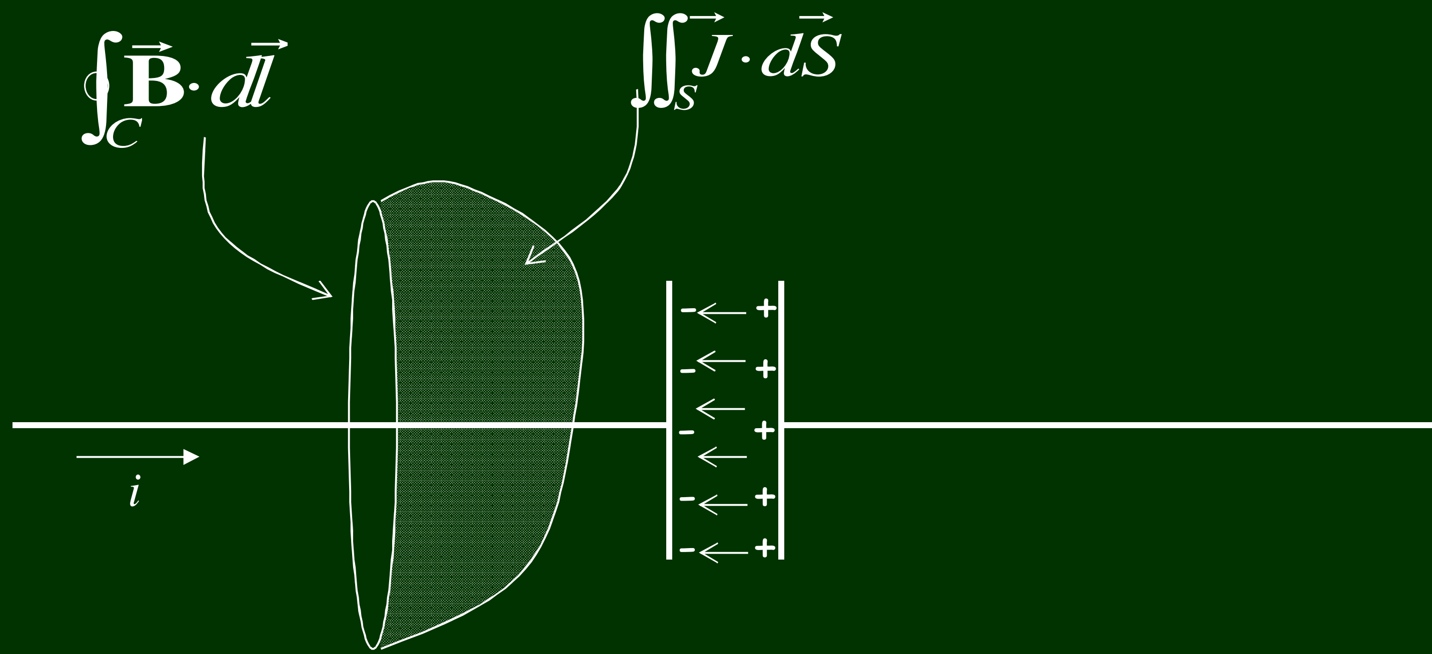
Ampere



Maxwell

"Nešto nedostaje..."

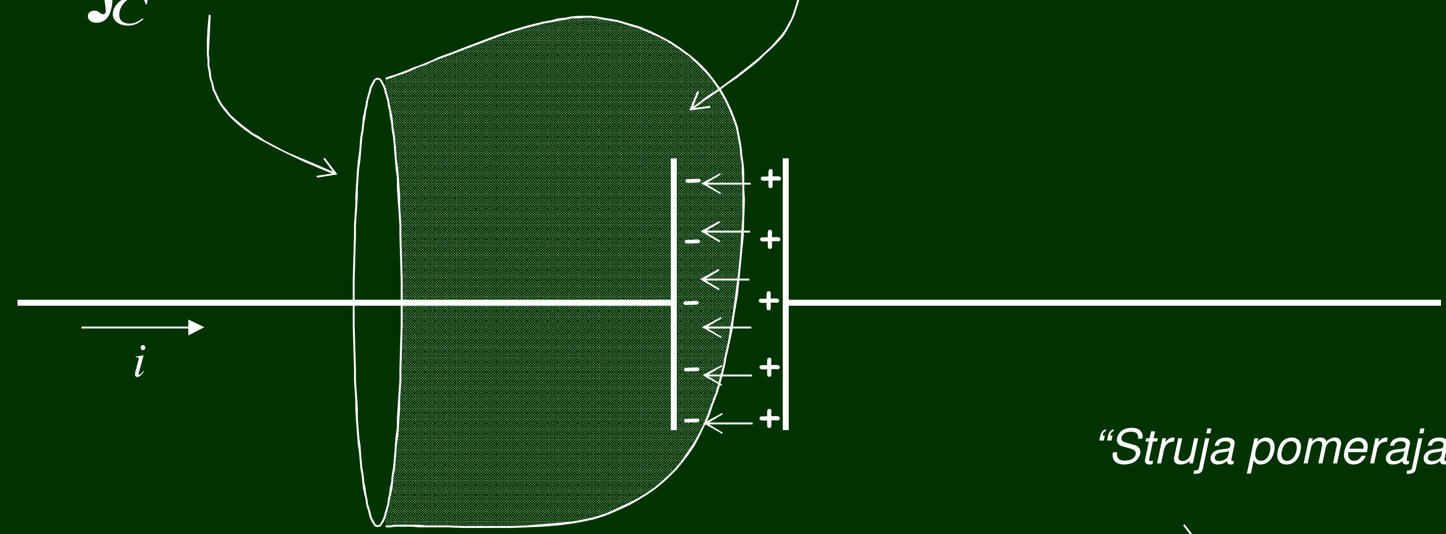
Punjenje kondenzatora



Punjenje kondenzatora

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$$

$$\iint_S \vec{J} \cdot d\vec{S}$$



"Struja pomeraja"

Maxwell: "…promena električnog polja u kondenzatoru je takođe struja."

$$\oint_C \vec{B} \cdot d\vec{l} = \mu \iint_S \left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}$$

MAKSVELOVE JEDNAČINE
(INTEGRALNA FORMA)

$$\oint_C \vec{E} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu \iint_S \left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}$$

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \iiint_V \rho \cdot dV$$

$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$

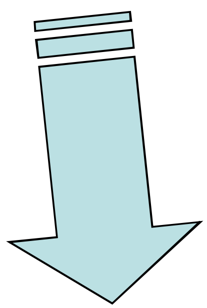
Maksvelovske jednačine u diferencijalnom obliku

Gausova teorema za divergenciju

$$\oiint_S \vec{F} \cdot \overrightarrow{dS} = \iiint_V \nabla \cdot \vec{F} dV$$

Stoksova teorema za rotor

$$\oint_L \vec{F} \cdot \overrightarrow{dl} = \iint_S \nabla \times \vec{F} \cdot d\vec{S}$$



$$\oint_L \vec{E} \cdot \vec{dl} = \iint_S \nabla \times \vec{E} \cdot d\vec{S}$$

$$\iint \nabla \times \vec{E} \cdot d\vec{S} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu \left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right)$$

$$\oiint_S \vec{E} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{E} dV$$

$$\iiint \nabla \cdot \vec{E} dV = \frac{1}{\varepsilon} \iiint \rho dV$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \cdot \vec{B} = 0$$